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Optimal Strategies Via Maximizing Net Present Values

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Abstract: *Project management is an important part of enterprise operation planning. Among numerous indexes of enterprise investment project evaluation, net present value is particularly accurate and conducive to rapid decision-making. Considering that in reality, due to the continuous expansion of the scale of the enterprise project, the investment cycle is becoming longer and longer, and the cash flow of the project occurs all the time, it seems more realistic to regard the cash flow as a continuous variable about time in the long run. Based on the constraints of initial principal and maximum investment dispersion, this paper studies the portfolio model of NPV maximization and NPV utility maximization, and gives the analytic solution of the optimal investment strategy.*

Keywords – *Duality, Geometric Brownian motion, HJB equation, Legendre transformation, Maximizing net present value*

I. Introduction

NPV refers to the difference between the present value of future cash inflow and the present value of future cash outflow calculated according to a certain expected discount rate. Among them, the expected discount rate is determined according to the lowest acceptable rate of return on investment, which is the lowest acceptable limit for an enterprise to make investment. Generally, a given rate of return greater than the cost of capital is chosen. NPV is the basic index to evaluate the investment project. If NPV is positive, the investment project can obtain positive return and the investment project is acceptable. The maximization of NPV means that when an enterprise has multiple projects to be invested, the enterprise hopes to obtain the maximum NPV of the investment project with the limited investment principal. At this time, it needs to consider the factors such as the rate of return and volatility of the investment project, and determine the share of the investment project, so as to realize the maximization of NPV under the given conditions.

For the expression of net present value, the traditional expression method of net present value contains a lot of assumptions, and these assumptions are often difficult to achieve in reality. Therefore, the research on the expression method of net present value at this stage focuses on relaxing these assumptions, so that the expression of net present value is based on some conditions more in line with the actual situation.

For the assumption of uncertainty of cash flow, many studies have introduced random factors to describe the uncertainty of cash flow in each period, so as to get a more realistic expression of NPV. For example, Gaspars-Wieloch and Helena [1] also studied the estimation of NPV under uncertainty, mainly introducing the uncertainty of cash flow frequency, As the inflow and outflow of cash are in the future, so the cash flow will naturally have uncertainty. In 2015, Leyman P and Vanhoucke M [2] studied the problem of maximizing the net present value under the condition of resource constraints, and used a new scheduling technology, which can optimize the maximum net present value by scheduling project activities. In 2016 [3], they further studied this problem and introduced investment capital constraint in the case of resource constraint, so the order of investment was also considered to prevent the cash flow from breaking during the project. In 2016, Rad M S, Jamili A, Tavakkoli-Moghaddam R and others [4] also conducted further research on the maximization of NPV under resource constraints. Wiesemann W, Kuhn D [5] discussed the problem of time stochastic net present value in their book published in 2015. They believe that to successfully manage capital intensive development and engineering projects, it is necessary to carefully arrange the timing of cash inflow and outflow.

However, most of the studies on the maximization of NPV only stay in the discrete state, and assume the inflow and outflow of cash flow occurring at regular intervals. But in fact, when the scale of the investment project becomes larger, if the cash inflow and outflow of the project is more frequent during the whole investment period, the cash flow is closer to continuous variable about time. Therefore, this paper hopes to optimize the traditional discrete cash flow through the continuous cash flow, and study the strategy of maximizing the net present value in the continuous case.

II. The maximum model of net present value

2.1 The construction of NPV maximization model

Assuming that the goal of the enterprise is to obtain the maximum NPV of the project on the basis of the given investment principal, taking the enterprise's investment limit and investment dispersion as the limiting conditions, this section constructs a continuous time model to maximize the NPV based on the limit of the total initial principal and investment concentration. This model can provide the first mock exam for the investment share of enterprises under multi project investment, and enable enterprises to make better use of limited investment capital to get greater return on investment.

We consider a portfolio of N investment projects, supposing that the initial time of the project is the present period (t=0), the amount invested into project i (i=1,2,…,N) is ω_i. In this way, the amounts invested across all projects can be expressed as $\vec{\omega}=(\omega_1, \omega_2, \ldots, \omega_N)^T \in R^N$. Each project i generates a continuous cash flow that is represented by $C_i(t)$ until the maturity period (t=T) of the project, and r is the interest rate.

Suppose that the cash flow $C_i(t)$ generated by project i at time t satisfies geometric Brownian motion, as shown below:

$$\begin{cases} dC_i(t) = C_i(t)(\mu_i dt + \sigma_i dW(t)) \\ C_i(0) = C_{i0} \end{cases} \tag{1}$$

where μ_i is the instantaneous expected rate of return of project i, σ_i is the instantaneous volatility of project i. Suppose that $\{W(t)|0 \le t \le T\}$ is a standard one-dimensional Brownian motion defined on the complete probability space (Ω, F, P). Suppose that the total cash flow at time t is C(t), then C(t) satisfies stochastic differential equation (SDE), as shown below:

$$\begin{cases} dC(t) = \sum_{i=1}^{N} \frac{\omega_i(0)C(t)}{C_i(t)} dC_i(t) \\ C(0) = c_0 \end{cases} \tag{2}$$

Then we can obtain the differential expression of C (T) is:

$$dC(t) = \sum_{i=1}^{N} \mu_i \omega_i(0)C(t)dt + \sum_{i=1}^{N} \sigma_i \omega_i(0)C(t)dW(t) \tag{3}$$

Under unconstrained conditions, the problem of maximizing NPV can be described as follows:

$$\begin{cases} \max\limits_{\vec{\omega}} E \int_0^T e^{-rt} C(t) dt \\ s.t. \begin{cases} dC(t) = \sum_{i=1}^{N} \mu_i \omega_i(0)C(t)dt + \sum_{i=1}^{N} \sigma_i \omega_i(0)C(t)dW(t) \\ C(0) = c_0 \end{cases} \end{cases} \tag{4}$$

This paper further adds two restrictions, namely, the restriction on the total amount of principal at the beginning of the period and the restriction on the degree of investment concentration:

$$\sum_{i=1}^{N} \omega_i(0) = A \tag{5}$$

$$\sum_{i=1}^{N} \omega_i^2(0) \le B \tag{6}$$

Using Lagrange multiplier method to add two constraints to the objective function, the problem of maximizing NPV based on constraints can be described as follows:

$$\begin{cases} \max_{\vec{\omega}} E\left\{\int_0^T e^{-rt}C(t)dt + \lambda\left(\sum_{i=1}^{N}\omega_i(0) - A\right) + \eta\left(\sum_{i=1}^{N}\omega_i^2(0) - B + \varepsilon(t)\right)\right\} \\ s.t. \begin{cases} dC(t) = \sum_{i=1}^{N}\mu_i\omega_i(0)C(t)dt + \sum_{i=1}^{N}\sigma_i\omega_i(0)C(t)dW(t) \\ C(0) = c_0 \end{cases} \end{cases} \quad (7)$$

2.2 The solution of NPV maximization model

In order to solve the above stochastic control problem, the HJB equation is used in this chapter. Firstly, the value function is defined:

$$H(t_0, c_0) = \max_{\vec{\omega}} E\left\{\int_{t_0}^{T} e^{-rt}C(t)dt\right\}, 0 < t < T$$

Then, according to the principle of dynamic programming, the original problem can be solved as follows:

$$\begin{aligned} H(t_0, c_0) &= \max_{\vec{\omega}} E\left\{\int_{t}^{T} e^{-rt}C(t)dt\right\} \\ &= \max_{\vec{\omega}} E\left\{\int_{t_0}^{t_0+\Delta t} e^{-rt}C(t)dt + \int_{t_0+\Delta t}^{T} e^{-rt}C(t)dt\right\} \\ &= \max_{t_0\le t\le t_0+\Delta t}{}_{\vec{\omega}} E\left\{\int_{t_0}^{t_0+\Delta t} e^{-rt}C(t)dt + \max_{t_0\le t\le t_0+T}{}_{\vec{\omega}} E\int_{t_0+\Delta t}^{T} e^{-rt}C(t)dt\right\} \\ &= \max_{t_0\le t\le t_0+\Delta t}{}_{\vec{\omega}} E\left\{\int_{t_0}^{t_0+\Delta t} e^{-rt}C(t)dt + H(t_0+\Delta t, c_0+\Delta c)\right\} \end{aligned} \quad (8)$$

where $c(t + \Delta t) = c_0 + \Delta c$.

According to the mean value theorem, there exists a point $\tilde{t}$ in $[t_0, t_0 + \Delta t]$ such that:

$$\int_{t_0}^{t_0+\Delta t} e^{-rt}C(t)dt = e^{-r\tilde{t}}\tilde{C}(t)\Delta t \quad (9)$$

here $\tilde{C}(t) = C(\tilde{t})$. Then let $\Delta t \to 0$, which is to say $\tilde{t} \to t_0$, we have:

$$\int_{t_0}^{t_0+\Delta t} e^{-rt}C(t)dt = e^{-rt_0}C(t_0)\Delta t \quad (10)$$

By substituting (10) into (9), we can get the following results:

$$H(t_0, c_0) = \max_{\vec{\omega}} E\{e^{-rt_0}C(t_0)\Delta t + H(t_0 + \Delta t, c_0 + \Delta c)\} \quad (11)$$

Because $H(t_0, c_0)$ is a quadratic continuous differentiable function, then according to Ito theorem:

$$\begin{aligned} H(t_0 + \Delta t, c_0 + \Delta c) &= H(t_0, c_0) + H_t\Delta t + H_c\Delta c + \frac{1}{2}\left(\sum_{i=1}^{N}\sigma_i\omega_i(0)c\right)^2 H_{cc}\Delta t \\ &= H(t_0, c_0) + \left(H_t + \sum_{i=1}^{N}\mu_i\omega_i(0)cH_c + \frac{1}{2}\left(\sum_{i=1}^{N}\sigma_i\omega_i(0)c\right)^2 H_{cc}\right)\Delta t \\ &\quad + \sum_{i=1}^{N}\sigma_i\omega_i(0)c\, H_c\Delta W(t) \end{aligned} \quad (12)$$

The expectation of Brownian motion in the (12), $E\left(\sum_{i=1}^{N}\sigma_i\omega_i(0)c\,H_c\Delta W(t)\right)=0$. Therefore, by substituting (12) into (11), we can get that:

$$H(t_0,c_0)=\max_{\vec{\omega}}E\left\{e^{-rt_0}C(t_0)\Delta t+H(t_0,c_0)+\left(H_t+\sum_{i=1}^{N}\mu_i\omega_i(0)cH_c+\frac{1}{2}\left(\sum_{i=1}^{N}\sigma_i\omega_i(0)c\right)^2H_{cc}\right)\Delta t+o(\Delta t)\right\} \quad (13)$$

Subtract $H(t_0,c_0)$ from both ends of (13), and divide by Δt. Let $\Delta t\to 0$, we can have:

$$\max_{\vec{\omega}}\left\{e^{-rt}C(t)+H_t+\sum_{i=1}^{N}\mu_i\omega_i(0)cH_c+\frac{1}{2}\left(\sum_{i=1}^{N}\sigma_i\omega_i(0)c\right)^2H_{cc}\right\}=0$$

Then the HJB equation can be obtained as follows:

$$H_t+\max_{\vec{\omega}}\left\{\sum_{i=1}^{N}\mu_i\omega_icH_c+\frac{1}{2}\left(\sum_{i=1}^{N}\sigma_i\omega_i\right)^2c^2H_{cc}\right\}=0 \quad (14)$$

Derivative of (14) with respect to ω_i, we can get:

$$\mu_icH_c+\sigma_i^2\omega_ic^2H_{cc}=0$$

The solution is:

$$\omega_i^*=-\frac{\mu_iH_c}{\sigma_i^2CH_{cc}}$$

After substituting ω_i^* into the limiting condition, we can get:

$$\frac{H_c}{CH_{cc}}=-\frac{B}{A\sum_{j=1}^{N}\frac{\mu_j}{\sigma_j^2}}$$

Hence, the analytic solution of ω_i^* is:

$$\omega_i^*=\frac{\mu_iB}{\sigma_i^2A\sum_{j=1}^{N}\frac{\mu_j}{\sigma_j^2}}$$

So far, we get the optimal investment strategy of maximizing the net present value in continuous time.

III. The utility maximization model of net present value

3.1 The construction of NPV utility maximization model

In this section, we consider that the goal of the enterprise is to maximize the utility of the total net present value of the project. By introducing the utility into the problem of net present value maximization, we can further consider the practical problem of the enterprise's risk attitude. In this section, we build a model for the problem of net present value utility maximization under constrained conditions.

In this paper, hyperbolic absolute risk aversion (HARA) utility function is used to describe the utility of enterprises, and its parameters include k, v and γ, the specific utility function is as follows:

$$U(c)=\frac{1-k}{vk}\left(\frac{v}{1-k}c+\gamma\right)^k,\quad c>0 \quad (15)$$

In this chapter, we assume that the objective function is the utility maximization of net present value:

$$\max_{\vec{\omega}} E\int_0^T e^{-rt}\frac{1-k}{vk}(\frac{v}{1-k}C(t)+\gamma)^k dt \tag{16}$$

Then the stochastic control problem can be described as follows:

$$\begin{cases} \max_{\vec{\omega}} E\left\{\int_0^T e^{-rt}\frac{1-k}{vk}\left(\frac{v}{1-k}C(t)+\gamma\right)^k dt+\lambda\left(\sum_{i=1}^N \omega_i(0)-A\right)+\eta\left(\sum_{i=1}^N \omega_i^2(0)-B+\varepsilon(t)\right)\right\} \\ s.t.\begin{cases} dC(t)=\sum_{i=1}^N \mu_i\omega_i(0)C(t)dt+\sum_{i=1}^N \sigma_i\omega_i(0)C(t)dW(t) \\ C(0)=c_0 \end{cases} \end{cases} \tag{17}$$

In order to solve the above stochastic control problem, the HJB equation is still used in this section. The calculation method is similar, and the HJB equation is obtained as follows:

$$H_t+\max_{\vec{\omega}}\left\{\sum_{i=1}^N \mu_i\omega_i cH_c+\frac{1}{2}\left(\sum_{i=1}^N \sigma_i\omega_i\right)^2 c^2H_{cc}\right\}=0 \tag{18}$$

Derivative of (18) with respect to ω_i, we can get:

$$\mu_i cH_c+\sigma_i^2\omega_i c^2H_{cc}=0$$

The solution is:

$$\omega_i^*=-\frac{\mu_i H_c}{\sigma_i^2 cH_{cc}} \tag{19}$$

The equation of value function H can be obtained by substituting ω_i^* back into (18):

$$H_t+\left[-\sum_{i=1}^N\frac{\mu_i^2}{\sigma_i^2}+\frac{1}{2}\left(\sum_{i=1}^N\frac{\mu_i}{\sigma_i}\right)^2\right]\frac{H_c^2}{H_{cc}}=0, \tag{20}$$

satisfies $H(T,c)=U(c)$.

According to the solution method of stochastic optimization control problem, we can see that when using HJB equation to solve the optimization strategy of portfolio problem, the solution of the initial problem can be transformed into solving (20). However, it is difficult to find the explicit solution of this differential equation, because (20) is a nonlinear differential equation and a second-order differential equation. Therefore, we need to transform (20) to make the equation easier to solve. Through Legendre transformation, it can be transformed into a more easily solved linear partial differential equation.

The following is the specific Legendre transformation process:

Definition 3.1 Let $f: R^n \to R$ be a convex function, and for a given $z>0$, then its Legendre transform is defined by:

$$L(z)=\max_c\{f(c)-zc\}, \tag{21}$$

Then the function $L(z)$ is called the dual function of Legendre transformation of $f(c)$.

If f(c) is a strictly convex function, then (21) has and only has a unique maximum point, so we might as well set it as c_0. The right side of the equal sign of (21) is used to calculate the derivative of c to obtain:

$$\frac{df(c)}{dc}-z=0,$$

Then we can have:

$$L(z)=f(c_0)-zc_0$$

If Legendre transformation is used, the premise is that the necessary condition is satisfied, that is, the value function is strictly convex. However, on the convexity of value function, Jonsson and Sircar [6] have proved that value function must be strictly convex on $t \in [0,T]$.

Then, according to the definition 3.1 of the above question, and using the convexity of the value function $H(t,c)$, we can define its Legendre transformation as follows:

$$\widehat{H}(t,z) = \max_{c>0}\{H(t,c) - zc | 0 < c < \infty\}, \ 0 < t < T, \tag{22}$$

where $z(z > 0)$ is the dual variable of c.

The optimal value of variable c can be obtained by $l(t,z)$ satisfying the following equation, where

$$l(t,z) = \min_{c>0}\{c | H(t,c) \geq zc + \widehat{H}(t,c)\}, \ 0 < t < T$$

To sum up, the following formula can be obtained

$$\widehat{H}(t,z) = H(t,l) - zl, \ l(t,z) = c, \tag{23}$$

According to (22) and (23), we can get $H_c = z$.

The first two parts of (23) are derived from z at the same time, we can obtain:

$$\widehat{H}_z = H_c \cdot l_z - l - zl_z = zl_z - l - zl_z = -l,$$

$$\Rightarrow l = -\widehat{H}_z.$$

The second order partial derivative of $\widehat{H}_z$ with respect to z is obtained:

$$\widehat{H}_{zz} = H_{cc}(l_z)^2 + H_c \cdot l_{zz} - l_z - l_z - zl_{zz} = H_{cc}(l_z)^2 + zl_{zz} - 2l_z - zl_{zz} = H_{cc}\widehat{H}_{zz}^2 + 2\widehat{H}_{zz}$$

Then we have:

$$H_{cc} = -\frac{1}{\widehat{H}_{zz}}$$

Similarly, by deriving both ends of $\widehat{H}(t,z) = H(t,l) - zl$ in (23) with respect to t at the same time, we can obtain:

$$\widehat{H}_t = H_t + H_c \cdot l_t - zl_t = H_t + zl_t - zl_t = H_t$$

In conclusion, we can obtain the derivative transformation rules between the value function $H(t,c)$ and its dual function $\widehat{H}(t,z)$, as follows:

$$H_c = z, \ \widehat{H}_t = H_t, \ H_{cc} = -\frac{1}{\widehat{H}_{zz}} \tag{24}$$

At the end of the project, we define:

$$\widehat{U}(z) = \max_{c>0}\{U(c) - zc | 0 < c < \infty\},$$

$$L(z) = \min_{c>0}\{U(c) \geq zc + \widehat{U}(z)\},$$

Then we can get the following results at the best point:

$$U'(c) - z = 0 \Rightarrow L(z) = c = (U')^{-1}(z)$$

Generally speaking, L is regarded as the inverse function of marginal utility function. According to $H(T,c) = U(c)$, at the end of the project, t=T, there is:

$$l(T,z) = \min_{c>0}\{c | U(x) \geq zc + \widehat{H}(T,c)\}$$

and

$$\widehat{H}(T,c) = \max_{c>0}\{U(c) - zc\}$$

hence, $l(T,z) = (U')^{-1}(z)$.

According to the derivative transformation rules of value function and its dual function summarized in (24), (19) can be further transformed into the following equation:

$$\widehat{H}_t + \left[\sum_{i=1}^{N}\frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N}\frac{\mu_i}{\sigma_i}\right)^2\right] z^2\widehat{H}_{zz} = 0, \tag{25}$$

Then, according to $c = l = -\hat{H}_z$, the derivative of both ends of (25) to z is obtained

$$\hat{H}_{tz} + \left[\sum_{i=1}^{N} \frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N} \frac{\mu_i}{\sigma_i}\right)^2\right]\left(2z\hat{H}_{zz} + z^2\hat{H}_{zzz}\right) = 0, \tag{26}$$

$$-l_t - \left[\sum_{i=1}^{N} \frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N} \frac{\mu_i}{\sigma_i}\right)^2\right](2zl_z + z^2 l_{zz}) = 0 \tag{27}$$

Through the above transformation, we have transformed the original equation. Through (27), we can see that the original equation has been transformed into a linear partial differential equation, which overcomes the inconvenience of finding explicit solutions for the nonlinear partial differential equation in (19).

Similarly, by substituting (24) into (19), we can get:

$$\omega_i^* = -\frac{\mu_i}{\sigma_i^2 l} z l_z, \tag{28}$$

From the expression of ω_i^*, we can see that through dual transformation, ω_i^* can be expressed as the expression of dual function l. In order to obtain the optimal investment strategy of maximizing the NPV of the project portfolio, we only need to continue to find the expression of l under the specific utility function, and then substitute it into (28).

3.2 The solution of utility maximization model of NPV

In this paper, we use HARA utility function to describe the utility of NPV:

$$U(c) = \frac{1-k}{vk}\left(\frac{v}{1-k}c + \gamma\right)^k, \quad c > 0$$

where $v > 0, k < 1, k \neq 0, \gamma > 0$, and $-\frac{\gamma(1-k)}{v} < c < \infty$.

According to $l(T,z) = (U')^{-1}(z)$, we can get

$$l(T,z) = \frac{1-k}{v}\left(z^{\frac{1}{k-1}} - \gamma\right).$$

Therefore, it can be speculated that the form of solution of (27) is

$$l(t,z) = \frac{1-k}{v}\left(z^{\frac{1}{k-1}} - \gamma\right)A(t) + B(t), \tag{29}$$

At the same time, the boundary conditions should be satisfied A(T)=1，B(T)=0.

According to (29), it can be concluded that:

$$\begin{cases} l_t = \frac{1-k}{v}\left(z^{\frac{1}{k-1}} - \gamma\right)A_t + B_t, \\ l_z = -\frac{A(t)}{v} z^{\left(\frac{1}{k-1}-1\right)}, \\ l_{zz} = \frac{A(t)}{v}\frac{k-2}{k-1} z^{\left(\frac{1}{k-1}-2\right)}. \end{cases} \tag{30}$$

By substituting (30) into (27), we can get that:

$$-\left(\frac{1-k}{v}\left(z^{\frac{1}{k-1}}-\gamma\right)A_t + B_t\right) + \left[\sum_{i=1}^{N} \frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N} \frac{\mu_i}{\sigma_i}\right)^2\right]\left(\frac{kA(t)}{v(k-1)} z^{\left(\frac{1}{k-1}\right)}\right) = 0 \tag{31}$$

According to the two boundary conditions mentioned above, A(T) = 1 and B(T) = 0, (31) can be transformed into solving the following two differential equations:

$$\begin{cases} B_t = 0 \\ B(T) = 0 \end{cases} \tag{32}$$

and

$$\begin{cases} -\frac{1-k}{v}\left(z^{\frac{1}{k-1}} - \gamma\right)A_t + \left[\sum_{i=1}^{N} \frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N} \frac{\mu_i}{\sigma_i}\right)^2\right]\left(\frac{kA(t)}{v(k-1)} z^{\left(\frac{1}{k-1}\right)}\right) = 0 \\ A(T) = 0 \end{cases} \tag{33}$$

By solving the differential (32) and (33), it is obtained that:

$$\begin{cases} A(t) = \exp\left\{ \dfrac{\left[\sum_{i=1}^{N}\frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N}\frac{\mu_i}{\sigma_i}\right)^2\right] k z^{\frac{1}{k-1}}}{(1-k)^2\left(z^{\frac{1}{k-1}} - \gamma\right)}(T-t) \right\} \\ B(t) = 0 \end{cases}$$

Therefore, we can get the expression of $l(t, z)$

$$l(t,z) = \frac{1-k}{v}\left(z^{\frac{1}{k-1}} - \gamma\right) \exp\left\{ \frac{\left[\sum_{i=1}^{N}\frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N}\frac{\mu_i}{\sigma_i}\right)^2\right] k z^{\frac{1}{k-1}}}{(1-k)^2\left(z^{\frac{1}{k-1}} - \gamma\right)}(T-t) \right\} \quad (34)$$

By substituting (34) back into (28), we get the following result:

$$\omega_i^* = \frac{\mu_i}{\sigma_i^2} \frac{\exp\left\{ \dfrac{\left[\sum_{i=1}^{N}\frac{\mu_i^2}{\sigma_i^2} - \frac{1}{2}\left(\sum_{i=1}^{N}\frac{\mu_i}{\sigma_i}\right)^2\right] k}{(1-k)^2}(T-t) \right\}}{(1-k)} \quad (35)$$

IV. Case analysis

On the basis of the theoretical research of the NPV utility maximization model based on the limitation of the total amount of initial principal and investment concentration in the third chapter, this chapter makes a further case analysis.

First of all, we assume that the number of projects available for selection is 5, that is, N = 5; the whole investment period is 10 years, that is, T = 10; assuming that the required minimum investment return rate, that is, the discount rate of project cash flow is 5%, that is, r = 5%; the total investment capital at the beginning of the period is 1 million, that is, A = 1 million. At the same time, we assume that the instantaneous expected returns of five projects are 6%, 7%, 8%, 9% and 10%, that is $\mu_1 = 6\%$, $\mu_2 = 7\%$, $\mu_3 = 8\%$, $\mu_4 = 9\%$, $\mu_5 = 10\%$; the instantaneous volatility of five projects is 10%, 11%, 12%, 13%, 14%, namely $\sigma_1 = 10\%$, $\sigma_2 = 11\%$, $\sigma_3 = 12\%$, $\sigma_4 = 13\%$, $\sigma_5 = 14\%$. For the parameters of utility function, we assume that k = 0.5, v = 0.5, γ = 0.5. The initial parameter of cash flow $c_0 = 10$.

4.1 The influence of project initial parameters on project NPV utility

In this section, the conditions of the initial instantaneous expected rate of return and the project discount rate given in the above cases are changed to analyze their impact on the maximum net present value utility of the project.

Taking the average instantaneous expected rate of return of the project as the horizontal axis and the maximum net present value utility of the project as the vertical axis, the influence curve of the average instantaneous expected rate of return on the maximum net present value utility of the project is drawn, as shown below:

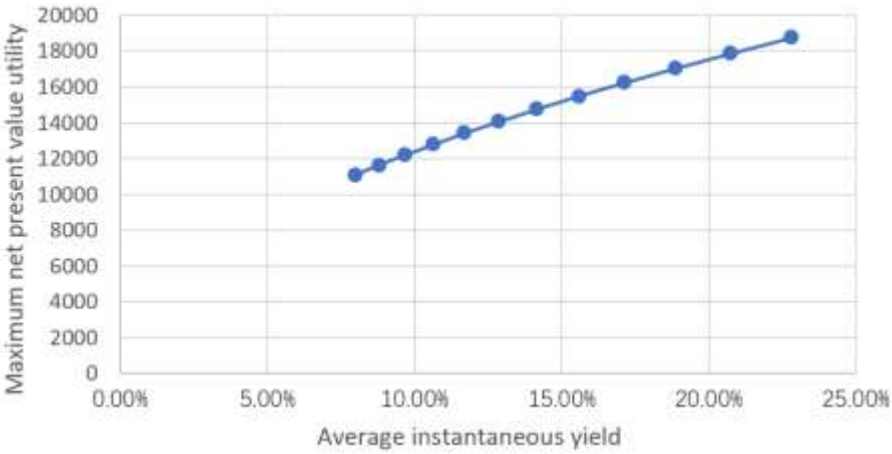

Figure 1: The influence of average expected rate of return on the utility of maximum net present value

It can be seen from Fig.1 that the overall maximum net present value utility of the project gradually increases with the increase of the average instantaneous expected rate of return of the project, indicating that the increase of the instantaneous expected rate of return of a single project will have a positive contribution to the overall maximum net present value utility, and the increase of the expected rate of return of the enterprise's investment project will increase the overall income of the enterprise.

At the same time, we can see that the slope of the curve is gradually decreasing, which indicates that the positive increase of the instantaneous expected rate of return on the overall NPV utility is marginal decreasing, which also conforms to the law of marginal utility decreasing.

At the same time, we still use a line chart to show the results, which makes the results more intuitive. Take the discount rate of the project, that is, the lowest acceptable rate of return, as the horizontal axis, and the maximum net present value utility of the project as the vertical axis, and draw the influence curve of the discount rate on the maximum net present value utility of the project, as shown below:

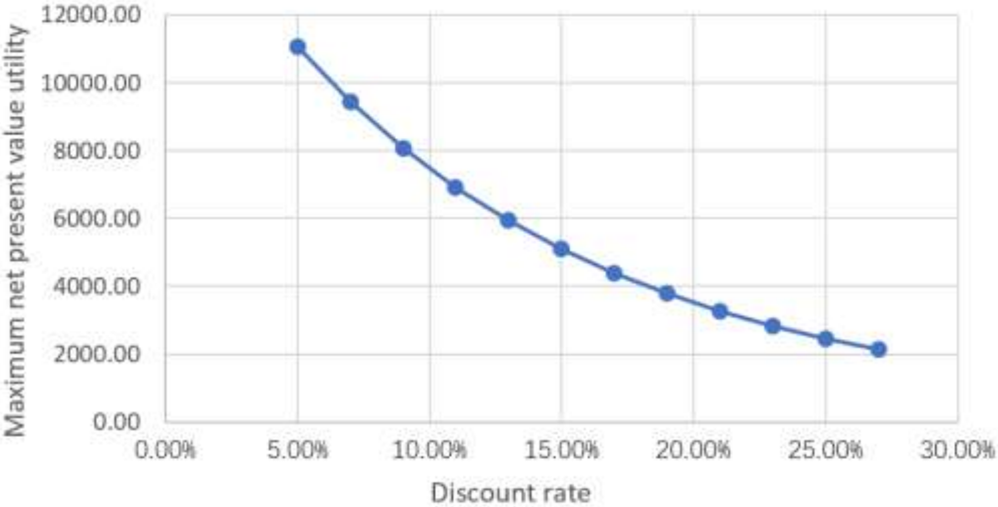

Figure 2: The influence of discount rate of return on the utility of maximum net present value

It can be seen from the above figure that the overall maximum net present value utility of the project decreases gradually with the increase of the minimum acceptable rate of return of the project, which indicates that the increase of the discount rate of the project will reduce the overall maximum net present value utility. This is because the discount rate reflects the time cost of capital and is the opportunity cost of enterprise investment. Therefore, the increase of the discount rate will reduce the enterprise investment project The total maximum net present value utility of.

At the same time, we can see that the slope of the curve is gradually decreasing, which indicates that the negative impact of the discount rate on the overall NPV utility is gradually weakening. When other conditions remain unchanged, the relationship between the two is similar to the inverse proportional function.

4.2 The influence of the parameters of utility function on the utility of NPV

The utility function used in this paper is $U(c) = \frac{1-k}{vk}(\frac{v}{1-k}c)^k$. We consider the impact on the total NPV utility of the project when the parameters v and k change.

First of all, we consider the impact on the overall NPV utility of the project when v changes. We still use the broken line chart to express the results, so that the results are more intuitive. Take the parameter V of the utility function as the horizontal axis, and the maximum net present value utility of the project as the vertical axis, and draw the influence curve of parameter v on the maximum net present value utility of the project, as shown below:

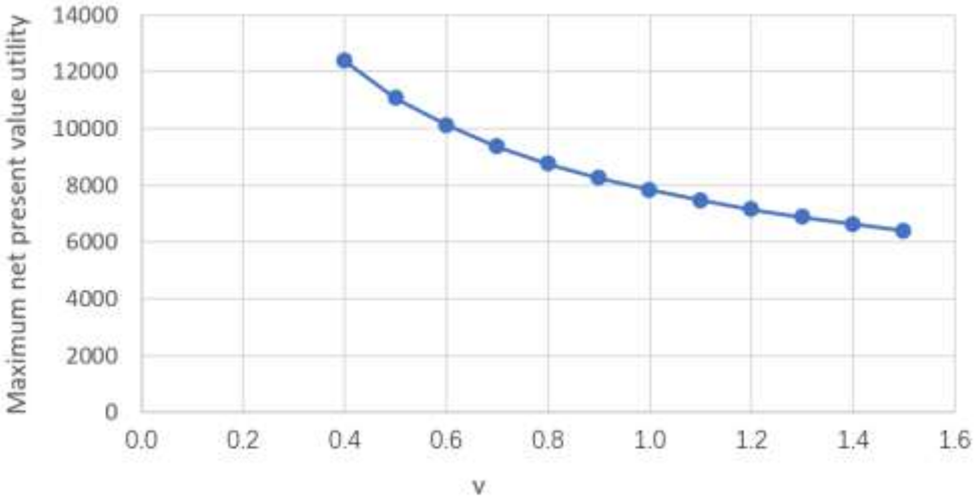

Figure 3: The influence of parameter v on the utility of maximum net present value

It can be seen that with the increase of v, the degree of risk preference of the investment enterprise is weakened, that is, it is more inclined to risk aversion, and the overall maximum net present value utility of the enterprise's project is declining. This shows that in the problem of net present value utility of the project, risk and income are still associated, and risk aversion will bring income, that is, the net present value utility will decline to a certain extent, and vice versa The same is true.

From the slope of the curve, we can see that the downward trend gradually slows down with the increase of v, which indicates that the decline degree of NPV utility brought by risk aversion is smaller and smaller; on the contrary, with the decrease of v, the increase of NPV utility brought by risk taking will be higher and higher, which can guide the trade-off between risk attitude and income of enterprises.

Secondly, we consider the impact on the overall NPV utility of the project when k changes. Taking the parameter k of the utility function as the horizontal axis and the maximum net present value utility of the project as the vertical axis, the influence curve of the parameter K on the maximum net present value utility of the project is drawn, as shown in the figure below:

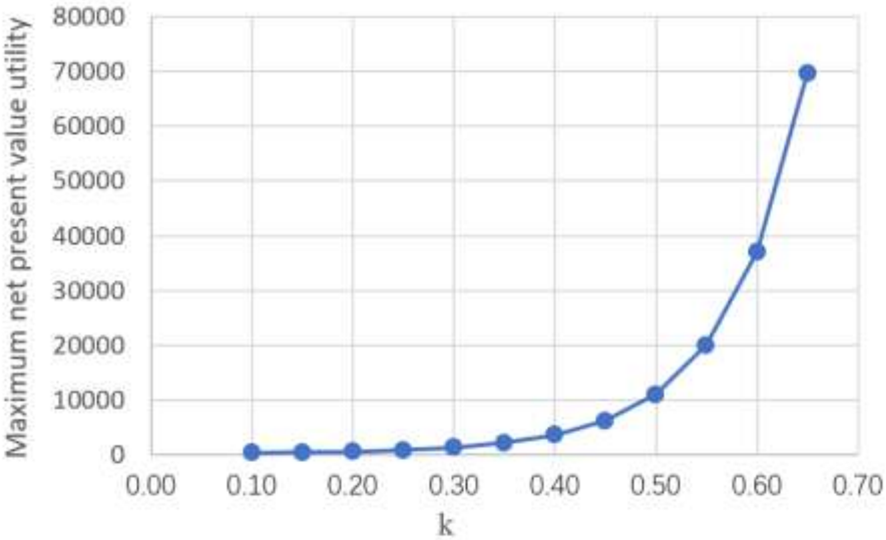

Figure 4: The influence of parameter k on the utility of maximum net present value

It can be seen that with the increase of K, the degree of risk aversion of investment enterprises decreases, that is, they are more inclined to risk-taking, and the overall maximum net present value utility of enterprises' projects is increasing, indicating that risk preference will bring benefits, that is, the net present value utility will increase, and vice versa.

From the slope of the curve, we can see that the rising trend increases with the increase of K, which indicates that the increase of NPV utility caused by risk-taking is more and more large, which is consistent with the analysis of parameter v.

4.3 Robustness analysis

In order to analyze the robustness of the model, this section will use other utility functions to analyze the original problem to test whether the NPV utility maximization model has differences.

Because the investment projects of enterprises are generally long-term investment, the time span is generally more than ten years or even decades, so enterprises will pay more attention to the long-term benefits of the project. As for the long-term utility function, the relevant research shows that the logarithmic utility function is more suitable than other utility functions. Therefore, this section uses the logarithmic utility function to describe the risk preference of enterprises, which is expressed as follows:

$$U(c) = \ln c, \quad c > 0$$

The utility maximization of net present value under logarithmic utility function is still solved by Legendre transformation. According to $l(T, z) = (U')^{-1}(z)$, we can get:

$$l(T, z) = \frac{1}{z}$$

V. Conclusion

Considering the large scale, long period and intensive cash flow of enterprise investment projects at present, the problem of maximizing net present value in discrete form is extended to the problem of maximizing net present value in continuous form, and the optimal investment strategy is given. This paper studies the expected utility maximization problem based on the initial net present value, and also gives the optimal investment strategy by using Legendre transformation method. These preliminary studies in this paper fill the gap of the research on the maximization of net present value in the field of corporate finance in the continuous case.

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