International Journal of Business Marketing and Management (IJBMM)
Volume 8 Issue 2 Mar-Apr 2023, P.P. 01-08
ISSN: 2456-4559
www.ijbmm.com

Research on Credit Default Swaps Pricing Under Uncertainty in the Distribution of Default Recovery Rates

Yuetong Fan[1], Peibiao Zhao*[1]
[1]*School of Mathematics & Statistics, Nanjing University of Science and Technology, China*
Email: 2863175374@qq.com;pbzhao@njust.edu.cn

***Abstract**: In this paper, we construct a Credit Default Swap pricing model for default recovery rates under distributional uncertainty based on a structured pricing model and distributional uncertainty theory. The model is algorithmically transformed into a solvable semi-definite programming problem using the Lagrangian dual method, and the solution of the model is given using the projection interior point method. Finally, an empirical analysis is conducted, and the results show that the model constructed in this paper is reasonable and efficient.*

Keywords: *Credit Default Swap, Default recovery rates, Distribution uncertainty, Structured pricing model*

I. Introduction

In recent years, Credit Default Swaps(CDS) have become one of the most active financial derivatives in the financial markets because of their good characteristics, and the pricing of Credit Default Swaps is one of the hot and difficult issues in the field of financial economics. Over the past 20 years or so, researchers have focused on solving for default probabilities and have obtained many important results, however, during the 2008 financial crisis, researchers found that default recovery rates have a profound impact on the pricing models of Credit Default Swaps. There are three general approaches to default recovery rates in the current industry: (1) Recovery of Face Value where creditors are compensated for a percentage of the value of the bond's face value following a debtor's default.(2)Recovery of Treasury i.e. after an event of default, the creditor is able to be compensated for a percentage of the value of a default-free Treasury bond that is equivalent to the bond, this percentage is usually set at 0.4. (3) Recovery of Market Value i.e. the default recovery is assumed to be a fraction of the market value of the bond prior to the event of default.

Such an approach, while easy, is unrealistic and can have a significant impact on the pricing model for credit derivatives.So in recent years some scholars have begun to model the distribution of default recovery rates, initially with Frye (2000) [1]who fitted the default recovery rate with a normal distribution.Pykhtin (2003) [2]improved Frye's model by applying a log-normal distribution to describe the default recovery rate.Rosch (2005) [3]proposed fitting the default recovery rate with a Logit-normal distribution. Subsequent research by Moody showed that real-life default recovery rates do not exhibit a simple single-peaked distribution, but rather a bimodal state, with recovery rates either in the vicinity of b eighty percent or in the vicinity of twenty percent, suggesting that the credit default swap prices obtained by choosing the mean of the default recovery rates in the model have a large deviation from the actual prices.In this way, the Beta distribution is more suitable for fitting the default recovery rate because it takes values in [0,1] and can make the probability density function curve show a double-peaked pattern by adjusting the two parameters in the density function, and later scholars mostly use this as the basis for modelling the default recovery rate,Chen(1999)[4] used the Beta distribution in constructing his credit risk model to fit the default recovery rate.Some scholars have also used the kernel function in non-parametric estimation to fit the default recovery.Brown (2004) [5]used the Beta-Bernstein polynomial smoothing technique to construct a smoothing kernelto fit the recovery density curve.

The Distributional Uncertainty Method is a method for dealing with parameters with uncertainty, which is solved using optimisation theory by constructing an uncertainty set containing all possible distributions of the parameters, constraining the uncertainty set, and then transforming the original problem into a robust problem corresponding to the uncertainty set.One of the earliest uses of uncertain stochastic models was by Scarf (1958) [6]who used a distribution uncertainty model to solve inventory control problems where only the mean and variance of demand were known. Delage and Ye (2010)[7] constructed distribution uncertainty sets for mean vectors and covariance matrices from historical data to study the loss function in best worst type

* Corresponding author

distribution robust optimization problems.Calafiore (2006)[8] studied the transformation of the problem into a quadratic cone constraint under conditions where only the mean and variance of the parameters are known, solving the linear chance control problem under conditions of uncertainty in the distribution.

This paper constructs a Credit Default Swap pricing model for default recovery rates under distributional uncertainty, mainly based on structured models and distributional uncertainty theory. Algorithmically, the Lagrangian dual method is used to transform the distributional robust chance constrained model into a solvable semi-definite programming problem, and the interior point projection method is used to give a solution, which has certain theoretical and practical significance.

The paper is structured as follows. In the second part, a model for pricing Credit Default Swaps under uncertainty in the distribution of default recovery rates is constructed based on a structured model. The third part is the empirical analysis, where we use Matlab to numerically calculate the price of Credit Default Swaps under uncertainty in the distribution of default recovery rates based on real-life cases of Credit Default Swaps and compare it with the actual price to verify the validity of the model. In the last section, we summarize the theories and models covered in this paper and give directions for further in-depth research on the Credit Default Swap problem based on distribution uncertainty.

II. Model formulation

In a credit default swap transaction, the purchaser of the default swap must pay a periodic fee (known as the credit default swap spread) to the seller of the default swap, usually at the end of every quarter, every six months or every year. In the event of a credit-type event (e.g. the bond host is unable to pay), the purchaser of the default swap has the right to demand full or partial compensation for losses from the seller of the default swap, and if no credit event occurs during the life of the contract, the seller of the default swap does not have to pay any money to the purchaser of the default swap and the contract terminates.

From the above description of the rationale for trading single-asset credit default swaps, we have the following pricing model: Consider a single-asset credit default swap contract with an underlying bond of face value V.The following are the necessary parameters we have defined.

(1)The expected recovery rate of the bond in the event of a default by the company is: R.

(2)The risk-free rate in the market is: r.

(3)The density of the firm's probability of default at any moment t during the term of the bond contract is: q(t).

(4)The annual premium to be paid by the purchaser of a default swap to the seller is: s

(5)The maturity date of the credit default swap contract is: T.

(6)The time of default of the enterprise occurs at the moment when: τ.

Assuming that premiums are paid quarterly, the premium payment moment is

$t_1 < t_2 < t_3 < \ldots < t_n = T$, so the amount of each premium payment is Δts.At the moment of default τ, the present value of all premiums payable by the purchaser of the default swap to the seller of the default swap is

$$s\left[\sum_{j=1}^{i} \Delta t e^{-r_j} + (\tau - t_i)e^{-rt_{j+1}}\right] = s[v_1(\tau) + v_2(\tau)] \tag{2.1}$$

The expectation of the present value of premiums paid by purchasers of default swaps is

$$\int_0^T q(\tau)s[v_1(\tau) + v_2(\tau)]\mathrm{d}\tau + \left(1 - \int_0^T q(\tau)\mathrm{d}\tau\right)sv_1(T) \tag{2.2}$$

At the same time, the present value of the payout to the purchaser of the default swap upon the occurrence of an event of default at time τ is $V(1-R)e^{-r\tau}$,Then the expectation of the present value of the payout is

$$\int_0^T q(\tau)V(1-R)e^{-r\tau}\mathrm{d}\tau \tag{2.3}$$

The pricing problem for credit default swaps refers to finding a "fair" price for the current contract before the underlying bond defaults, so our objective is to match the payout at default as closely as possible to the present value of the premium paid by the purchaser of the contract, i.e. to minimise the hedge. Therefore, the objective function in this paper is based on the minimisation of hedging as follows

$$\min_s \left(\int_0^T q(\tau)V(1-R)e^{-r\tau}\mathrm{d}\tau\right) - \left(\int_0^T q(\tau)s[v_1(\tau) + v_2(\tau)]\mathrm{d}\tau + \left(1 - \int_0^T q(\tau)\mathrm{d}\tau\right)sv_1(T)\right) \tag{2.4}$$

Because of the excellent characteristics of the bimodal Beta distribution, most academics currently use it to fit the default recovery rate, but comparative studies have found that the Beta distribution is very sensitive to parameters, and the robustness of the model based on the Beta distribution is poor. The true default recovery rate is not far from the value obtained from the Beta distribution. Therefore, we use the default recovery rate based on the Beta distribution as the criterion to make a CDS pricing model under the uncertainty of the default recovery rate distribution.The following assumptions are made first.

Definition 1. *Random variable R in the asymmetric uncertainty set* $\mathbb{D}_F(\nu_1,\nu_2)$

$$\mathbb{D}_F(\nu_1,\nu_2)=\left\{P\in\mathcal{M}+:\begin{array}{l}P(R\in\Omega)=1,\\ |\mu(R)-\hat{\mu}|\le\nu_1,\\ |\hat{\sigma}-\sigma(R)|\le\nu_2,\sigma(R)\succ 0\end{array}\right\}. \tag{2.5}$$

Here

$$\widehat{\sigma}=\frac{1}{N-1}\sum_{\mathrm{i=1}}^{N}(R^{[\mathrm{i}]}-\widehat{\mu})\left(R^{[\mathrm{i}]}-\widehat{\mu}\right)^T \tag{2.6}$$

$$\widehat{\mu}=\frac{1}{N}\sum_{\mathrm{i=1}}^{N}R^{[\mathrm{i}]} \tag{2.7}$$

The parameters ν_1,ν_2 control the size of the uncertainty set and the degree of uncertainty We make the following chance constraints.

$$\inf_{R\in\mathbb{D}_F(\nu_1,\nu_2)}P\{(R-R')^2<b\}\ge 1-\epsilon \tag{2.8}$$

where R' is the default recovery rate estimated from the bimodal Beta distribution, b is the degree of investor distrust in the bimodal Beta distribution, and $1-\epsilon$ is the probability of safety, given in advance by the investor.

For the chance constraint (2.8), the following proposition can be equivalently transformed into the form of a matrix inequality, and in combination with the objective function (2.4) , the initial distributionally robust optimization model with an uncertain set of uncertain parameters can eventually be transformed into a computationally solvable semi-definite programming problem. Compared to traditional pricing models, this is more realistic and more in line with the actual characteristics of the market.

Theorem 2.1. *The chance constraint with the set* $\mathbb{D}_F(\nu_1,\nu_2)$ *as an uncertain set is equivalent to the set of inequalities in the following equation.*

$$\begin{aligned}&\vartheta+A\bar{\mu}+B(\bar{\mu}^2+\bar{\sigma}^2)<1-\epsilon;\\&\begin{bmatrix}\vartheta-1 & \frac{A}{2}\\ \frac{A}{2} & B\end{bmatrix}\le 0;\\&\begin{bmatrix}\vartheta-Mb+MR'^2 & \frac{1}{2}(A-2MR')\\ \frac{1}{2}(A-2MR') & M+B\end{bmatrix}\le 0;\\&\vartheta,A,B\in\mathcal{R};M\ge 0\end{aligned} \tag{2.9}$$

Proof. The default recovery rate R may be a discrete random variable or a continuous random variable, which is treated as a continuous random variable in this paper in order not to lose generality.

For ease of analysis, we split the uncertainty set $\mathbb{D}_F(\nu_1,\nu_2)$.We defines

$$\mathcal{U}_{(\bar{\mu},\bar{\sigma})_F}=\{(\mu,\sigma)\in\mathbb{R}\times S_+ \mid \mu-\hat{\mu}\mid\le\nu_1,|\sigma-\hat{\sigma}|\le\nu_2\}. \tag{2.10}$$

and defines a family of distributions $\mathcal{F}$ as

$$\mathcal{F}=\{\mathrm{P}\in\mathcal{M}_+\mid \mathrm{P}(R\in\Omega)=1,\mathrm{E_P}(R)=\bar{\mu},D_\mathrm{P}(R)=\bar{\sigma}\} \tag{2.11}$$

Thus for the left-hand side of the chance constraint inequality (2.8) there is an equivalent form as follows.

$$\begin{aligned} &\inf_{(\bar{\mu},\bar{\sigma})\in\mathcal{U}_{(\hat{\mu},\hat{\sigma})_F}} \inf_{p\in\mathcal{F}} && \int_{\mathcal{R}} I_{(-\infty,0]}\{(R-R')^2-b\}dF(R) \\ &\text{s.t.} && \int_{\mathcal{R}} d\,F(R)=1, \\ & && \int_{\mathcal{R}} R\,dF(R)=\bar{\mu}, \\ & && \int_{\mathcal{R}} R^2\,dF(R)=\bar{\sigma}^2+\bar{\mu}^2 \\ & && dF(R)\geq 0. \end{aligned} \quad (2.12)$$

For $\inf_{p\in\mathcal{F}} \int_{\mathcal{R}} \mathrm{I}_{(-\infty,0]}\{(\mathrm{R}-\mathrm{R}')^2-\mathrm{b}\}\mathrm{dF}(\mathrm{R})$ and its constraints, the following Lagrangian dual form can be obtained.

$$\begin{aligned} \max_{\vartheta,A,B\geq 0} \inf_{dF(R)\geq 0} \vartheta\left(1-\int dF(R)\right)+A\left(\bar{\mu}-\int RdF(R)\right)+ \\ B\left(\bar{\sigma}^2+\bar{\mu}^2-\int R^2dF(R)\right)+\int 1_{(-\infty,0]}\{(R-R')^2-b\}dF(R). \end{aligned} \quad (2.13)$$

Further organizing (2.13) yields

$$\begin{aligned} \max_{\vartheta,A,B\geq 0} \vartheta+A\bar{\mu}+B(\bar{\mu}^2+\bar{\sigma}^2)+\inf_{dF(R)\geq 0} \\ \int \left(1_{(-\infty,0]}\{(R-R')^2-b\}-\vartheta-AR-BR^2\right)F(R). \end{aligned} \quad (2.14)$$

where ϑ, Λ, B are Lagrange multipliers. Given ϑ, Λ, B to obtain the minimum of the inner function, the product function in equation (2.14) must be non-negative, otherwise the minimum will be taken to negative infinity, so for any random variable R we have

$$1_{(-\infty,0]}\{(R-R')^2-b\}-\vartheta-AR-BR^2\geq 0 \quad (2.15)$$

After sorting, equation (2.15) is transformed into the following programming problem with constraints.

$$\max_{\vartheta,A,B\geq 0} \vartheta+A\bar{\mu}+B(\bar{\mu}^2+\bar{\sigma}^2) \quad (2.16)$$

$$\text{st.}1_{(-\infty,0]}\{(R-R')^2-b\}\geq\vartheta+AR+BR^2,\forall R \quad (2.17)$$

where the feasible domain of the constraint is

$$\vartheta+AR+BR^2\leq 1 \quad (2.18)$$

$$\vartheta+AR+BR^2\leq 0,\forall R:(R-R')^2-b>0 \quad (2.19)$$

Equation (2.18) can be equivalently translated into the following matrix inequality form.

$$\begin{bmatrix} \vartheta-1 & \frac{A}{2} \\ \frac{A}{2} & B \end{bmatrix}\leq 0 \quad (2.20)$$

Equation (2.19) can be equated as

$$\max\{\vartheta+AR+BR^2:(R-R')^2-b>0\}\leq 0 \quad (2.21)$$

The equivalent pairwise form to that shown in equation (2.20) is

$$\min_{M\geq 0}\max_{R}\{\vartheta+AR+BR^2+M\{(R-R')^2-b\}\}\leq 0 \quad (2.22)$$

Equation (2.22) is equivalent to $\exists\mathrm{M}\geq 0$ such that $\forall\mathrm{R}$ satisfies the following condition.

$$c\vartheta-Mb+MR'^2+(A-2MR')R+(M+B)R^2\leq 0 \quad (2.23)$$

$$\Leftrightarrow \exists M\geq 0,\text{s.t.}\begin{bmatrix} \vartheta-Mb+MR'^2 & \frac{1}{2}(A-2MR') \\ \frac{1}{2}(A-2MR') & M+B \end{bmatrix}\leq 0. \quad (2.24)$$

As evidenced above, the left-hand side of the chance constraint (2.8) is equivalent to equation (2.12),For $\inf_{p\in\mathcal{F}} \int_{\mathcal{R}} \mathrm{I}_{(-\infty,0]}\{(\mathrm{R}-\mathrm{R}')^2-\mathrm{b}\}\mathrm{dF}(\mathrm{R})$ and its constraints can be transformed by Lagrangian methods into an optimization problem with equation (2.20) and equation (2.24)as constraints and equation (2.4) as the objective function of the optimization problem, so that the chance constraint equation (2.8) is equivalent to equation (2.25) below as well as equations (2.20) and (2.24).

$$\inf_{(\bar{\mu},\bar{\sigma})\in\mathcal{U}_{(\bar{\mu},\bar{\sigma})_F}} \max_{\vartheta,A,B} \vartheta + A\bar{\mu} + B(\bar{\mu}^2+\bar{\sigma}^2) < 1-\epsilon \tag{2.25}$$

From equation (4.16), the mean and variance under $(\bar{\mu},\bar{\sigma}) \in \mathbb{R}\times S_+$ satisfy

$$|\bar{\mu}-\hat{\mu}| \le \nu_1 \tag{2.26}$$

$$|\bar{\sigma}-\hat{\sigma}| \le \nu_2, \bar{\sigma} > 0 \tag{2.27}$$

that is

$$\hat{\mu}-\nu_1 \le \bar{\mu} < \hat{\mu}+\nu_2 \tag{2.28}$$

$$0 < \bar{\sigma} < \hat{\sigma}+\nu_2 \tag{2.29}$$

Bringing equation (2.28), (2.29) into equation (2.25), equation (4.32) is equivalent to

$$\exists\vartheta, A, B \ge 0, \quad \text{s.t.}\,\vartheta + A\bar{\mu} + B(\bar{\mu}^2+\bar{\sigma}^2) < 1-\epsilon \tag{2.30}$$

The Lagrangian dyadic approach, combined with the objective function (2.4), leads to the following pricing model.

$$\begin{aligned}
\min_s \quad & \left(\int_0^T q(\tau)V(1-R)e^{-r\tau}\mathrm{d}\tau\right) - \left(\int_0^T q(\tau)s[v_1(\tau)+v_2(\tau)]\mathrm{d}\tau + \left(1-\int_0^T q(\tau)\mathrm{d}\tau\right)sv_1(T)\right) \\
\text{s.t.} \quad & \vartheta + A\bar{\mu} + B(\bar{\mu}^2+\bar{\sigma}^2) < 1-\epsilon; \\
& \begin{bmatrix} \vartheta-1 & \frac{A}{2} \\ \frac{A}{2} & B \end{bmatrix} \le 0; \\
& \begin{bmatrix} \vartheta - Mb + MR'^2 & \frac{1}{2}(A-2MR') \\ \frac{1}{2}(A-2MR') & M+B \end{bmatrix} \le 0; \\
& \vartheta, A, B \in \mathcal{R}; M \ge 0
\end{aligned} \tag{2.31}$$

III. Empirical analysis

In this chapter, we use one credit risk mitigation warrant in the market for our empirical analysis: the 18 Origin Water cp002 Joint Credit risk Mitigation warrant.

To investigate the effect of the parameters of the uncertainty set on the price of the credit default swap pricing model, the following parameters are defined Number.

$$\mathrm{R_x} = \frac{\mathrm{x}^*(\nu_1,\nu_2) - \mathrm{x}^*(\hat{\nu}_1,\hat{\nu}_2)}{\mathrm{x}^*(\hat{\nu}_1,\hat{\nu}_2)} \tag{3.1}$$

where $\mathrm{x}^*(\nu_1,\nu_2)$ and $\mathrm{x}^*(\hat{\nu}_1,\hat{\nu}_2)$ denote the prices of credit default swaps under (ν_1,ν_2) and $(\hat{\nu}_1,\hat{\nu}_2)$. Consider the parameters of the ambiguous set varying in a range of 20 %, i.e.

$$\frac{|\nu_i - \hat{\nu}_i|}{\hat{\nu}_i} \le 20\%, i = 1,2 \tag{3.2}$$

According to the Bootstrapping method, the estimates of ν_1 and ν_2 can be obtained as

$$\hat{\nu}_1 = 0.0564$$
$$\hat{\nu}_2 = 0.0332 \tag{3.3}$$

Then the parameters ν_1 and ν_2 vary in the range $[0.046730,0.06935] \times [0.02750,0.03989]$ and the result is shown below

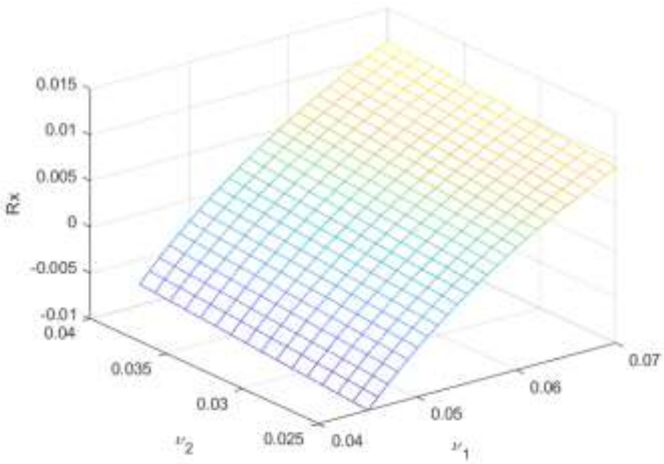

Figure 1. The effect of ambiguous set parameters on CDS prices

As can be seen from the graph, the prices of CDS are more sensitive to changes in ν_1 and change faster in the ν_1 direction, correspondingly the prices of CDS are less sensitive to changes in ν_2 and the slope of credit default swap prices in the ν_1 direction is significantly greater than the slope in the ν_2 direction.

We use the linear matrix inequality toolkit in Matlab to solve the semi-definite programming problem shown in equation (2.31), where we set the investor's distrust of the bimodal Beta to 0.1, i.e. b = 0.1, the risk-free return is set to 0.035, and the probability of investor distrust of the bimodal Beta distribution ϵ is set to 0.05, and the results obtained are shown in Fig2.

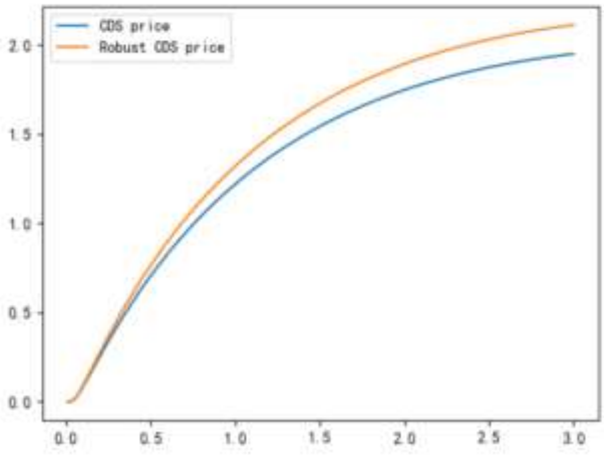

Figure 2. The effect of parameter γ_1 on the optimal strategy π^*

The blue curve in the graph indicates the price of a robust CDS when the distribution of default recovery rates is uncertain, while the red curve indicates the price of a CDS when default recovery rates follow a bimodal Beta distribution. It is clear that the price of a robust credit default swap lies below the price of a credit default swap under the bimodal Beta distribution, indicating that the bimodal Beta distribution overestimates the price of a credit default swap, which can result in larger losses to investors in the event of a bias in the default recovery rate.

This is next compared to the actual price, and the stock price for Origin Water from November 19, 2018 to November 19, 2019 is chosen for this paper, and then estimates of other parameters are obtained based on previous research, with the following results.

Measurement results

CDS prices under a bimodal Beta distribution	CDS prices under distributional uncertainty	Market offer
1.32	1.11	1.08

IV. Summary and prospect

In the pricing of Credit Default Swaps, the estimated value of the default recovery rate is obtained by estimating the probability distribution assumed in advance, but in real markets, the distribution of default recovery rates is affected by a variety of factors and investors are often unable to use the limited information available to obtain the exact probability distribution of default recovery rates. It is based on these practical factors that we introduce uncertain distributions into portfolios and develop a pricing model for credit default swaps under uncertain distributions of default recovery rates. Finally, based on Matlab numerical calculations, we solve for the prices of Credit Default Swaps under uncertain distributions in conjunction with actual cases, giving optimal results and comparing them with actual results to verify the validity of the model.

Due to the limitations of the author's own professional level, the problems studied in this paper still have shortcomings and need to be improved: for example, when constructing the distribution uncertainty set, higher-order and more refined uncertainty sets can be introduced; this paper assumes that the default probability and default recovery rate are independent of each other for the sake of computational simplicity, and the correlation between these two random variables can be considered; in addition, a more complex process of enterprise asset value can be considered. For the above shortcomings, we hope that future scholars will do further research.

Reference

[1] Frye J. Depressing recoveries[J].Risk London Risk Magazine, 2000, 13(11): 108-111.

[2] Pykhtin M.Recovery rates: unexpected recovery risk[J]. Risk London Risk Magazine, 2003, 16(8): 74-79.

[3] Rösch D, Scheule H. A multi-factor approach for systematic default and recovery risk[J].The Journal of Fixed Income, 2011: 117-135.

[4] Chen, Song Xi. Beta kernel estimators for density functions[J].Computational Statistics and Data Analysis,1999,31: 131-145.

[5] Brown B M,Chen S X.On the way to recovery:A nonparametric bias free estimation of recovery rate densities[J].Journal of Banking and Financial,2004,28(12):2915-2931.

[6] Scarf H E. A min-max solution of an inventory problem[M]. Santa Monica: Rand Corporation, 1957.

[7] Delage E, Ye Y. Distributionally robust optimization under moment uncertainty with application to data-driven problems[J]. Operations Research, 2010, 58(3): 595-612.

[8] Calafiore G C, Ghaoui L E. On distributionally robust chance-constrained linear programs[J]. Journal of Optimization Theory and Applications, 2006, 130(1): 1-22.

[9] Zhilin Kang, Xun Li, Zhongfei Li & Shushang Zhu (2019) Data-driven robust mean-CVaR portfolio selection under distribution ambiguity, Quantitative Finance, 19:1, 105-121.

[10] Kang Z, Li X, Li Z, et al. Data-driven robust mean-CVaR portfolio selection under distribution ambiguity[J]. Quantitative Finance, 2019, 19(1): 105-121.

[11] Friedman C, Sandow S. Model performance measures for expected utility maximizing investors[J]. International Journal of Theoretical and Applied Finance, 2003, 6(04): 355-401.

[12] Cox J C, Huang C F. Optimal consumption and portfolio policies when asset prices follow a diffusion process[J]. Journal of Economic Theory, 1989, 49(1):33-83.

[13] Liang J, Zou H. Valuation of credit contingent interest rate swap with credit rating migration[J]. International Journal of Computer Mathematics, 2020, 97(12): 2546-2560.

[14] Zhang Y, Gao J, Fu Z. Valuing currency swap contracts in uncertain financial market[J]. Fuzzy Optimization and Decision Making, 2019, 18(1): 15-35.

[15] Black F, Cox J C. Valuing corporate securities: some effects of bond indenture provisions[J]. The Journal of Finance, 1976, 31(2): 351-367.

[16] Merton R C. On the pricing of corporate debt: The risk structure of interest rates[J]. The Journal of Finance, 1974, 29(2): 449-470.

[17] Shimko D C, Tejima N, Van Deventer D R. The pricing of risky debt when interest rates are stochastic[J]. The Journal of Fixed Income,1993,3(2):58-65.

[18] Wang A. The pricing of total return swap under default contagion models with jump-diffusion interest rate risk[J]. Indian Journal of Pure and Applied Mathematics, 2020, 51(1): 361-373.

[19] Chen, L., He, S. and Zhang, S., Tight bounds for some risk measures, with applications to robust portfolio selection. Oper. Res., 2011, 59, 847–865.

[20] Jarrow R A, Yu F. Counterparty risk and the pricing of defaultable securities[J]. The Journal of Finance, 2001, 56(5): 1765-1799.

[21] Liang J, Zou H. Valuation of credit contingent interest rate swap with credit rating migration[J]. International Journal of Computer Mathematics, 2020, 97(12): 2546-2560.

[22] Zhang Y, Gao J, Fu Z. Valuing currency swap contracts in uncertain financial market[J]. Fuzzy Optimization and Decision Making, 2019, 18(1): 15-35.

[23] Marple T. The social management of complex uncertainty: Central Bank similarity and crisis liquidity swaps at the Federal Reserve[J]. The Review of International Organizations, 2021, 16(2): 377-401.

[24] Jansen J, Das S R, Fabozzi F J. Local volatility and the recovery rate of credit default swaps[J]. Journal of Economic Dynamics and Control, 2018, 92: 1-29.

[25] Siao J S, Hwang R C, Chu C K. Predicting recovery rates using logistic quantile regression with bounded outcomes[J]. Quantitative Finance, 2016, 16(5): 777-792.

[26] Lin J H, Chen S, Huang F W. A contingent claim model of life insurer-bank swap default pricing: strategic substitutes and complements[J]. Applied Economics, 2021, 53(36): 4166-4177.

[27] Peel D, McLachlan G J. Robust mixture modelling using the t distribution[J]. Statistics and Computing, 2000, 10(4): 339-348.

[28] Jiang R, Guan Y. Data-driven chance constrained stochastic program[J]. Mathematical Programming, 2016, 158(1): 291-327.

[29] Mohajerin Esfahani P, Kuhn D. Data-driven distributionally robust optimization using the Wasserstein metric: Performance guarantees and tractable reformulations[J]. Mathematical Programming, 2018, 171(1): 115-166.

[30] Kim S I, Kim Y S. Tempered stable structural model in pricing credit spread and credit default swap[J]. Review of Derivatives Research, 2018, 21(1): 119-148.