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# Robust Optimal Reinsurance and Investment Problem with p-Thinning Dependent and Default risks

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**Abstract:** *In this paper, we consider an AAI with two types of insurance business with p-thinning dependent claims risk, diversify claims risk by purchasing proportional reinsurance, and invest in a stock with Heston model price process, a risk-free bond, and a credit bond in the financial market with the objective of maximizing the expectation of the terminal wealth index effect, and construct the wealth process of AAI as well as the the model of robust optimal reinsurance-investment problem is obtained, using dynamic programming, the HJB equation to obtain the pre-default and post-default reinsurance-investment strategies and the display expression of the value function, respectively, and the sensitivity of the model parameters is analyzed through numerical experiments to obtain a realistic economic interpretation. The model as well as the results in this paper are a generalization and extension of the results of existing studies.*

**Key words:** *p-Thinning dependent risk, defaultable bonds, dynamic programming, HJB equation, robust optimization.*

## I. Introduction

The sound operation and solid claiming ability of insurance companies are important to maintain social stability and promote social development, and the purchase of reinsurance can well control the risk of insurance companies. The current research on reinsurance in the actuarial field of insurance mainly focuses on the insurer's objective function, investment type, claim process, and price process of assets. [1]-[3]studied the most available reinsurance investment problems under different utility function. [4]-[7]investigate the optimal reinsurance investment problem under different risky asset price process.

Most of the current literature considers only single-risk claims, while in reality the risks are usually correlated with each other. The current literature on claim dependency risk is divided into two main types of dependency, one is co-impact and the other is p-thinning dependency. P-thinning dependency means that when one type of claim occurs, there is a certain probability p that another type of claim will occur, for example, a car accident or fire will not only cause property damage, but also loss of life if the situation is serious. The problem under diffusion approximation model, jump diffusion risk model is studied in[8][9]， [10] studied the type of dependent risk for common shocks, and [11] considered the risk process under p-thinning dependence. Most of the current studies consider only two assets for considering investments in credit bonds. [12]-[14] take into account defaulted bonds among the types of investments for insurers. Since models with diffusion and jump terms usually introduce uncertainty in the model for insurers, insurers usually want to seek a more robust model. [15]-[17] study the AAI most reinsurance-investment problem.

Based on this, this paper studies the optimal proportional reinsurance-investment problem of AAI under the Heston model by considering both sparse dependence and claims risk based on [11][14][17]. The paper is organized as follows: a robust optimal reinsurance-investment problem model under p-thinning dependence and default risk is developed in Section 2. The value function is divided into pre-default and post-default in Section 3, and the optimal reinsurance-investment strategies are solved for pre-default and post-default using dynamic programming, optimal control theory, and the HJB equation, respectively. Parameter sensitivities are analyzed, and economic explanations are given in Section 4. Section 5 concludes the paper.

## II. Model Formulation

Suppose $\{\Omega, \mathrm{F}, \{F_t\}_{t\in[0,T]}, \mathrm{P}\}$ is a complete probability space, the positive number T denotes the final value moment, [0, T ] is a fixed time interval, $F_t$ denotes the information in the market up to time t, and $\mathbb{F} := \{F_t\}_{t\in[0,T]}$ denotes the standard Brownian motion $W_0(t), W_1(t), W_2(t), W_3(t)$. Poisson process N(t), and the right-continuous P-complete information flow generated by the sequence of random variables $\{X_i, i \geq 1\}$, $\{Y_i, i \geq 1\}$. $\mathbb{H} := (\mathcal{H}_t)_{t\geqslant 0}$ is the information flow generated by the violation process H(t), let $\mathbb{G} := (\mathcal{G}_t)_{t\geqslant 0}$, be the information flow generated by $\mathbb{F}, \mathbb{H}$ the expanded information flow, i.e., $\mathbb{G} := \mathcal{F}_t \vee \mathcal{H}_t$. By definition, each $\mathbb{F}$ -harness is also the $\mathbb{G}$ -harness. The probability measure P is a realistic probability measure and Q is a risk-neutral measure. In addition, it is assumed that all transactions in the financial market are continuous, and no taxes do not incur transaction costs and all property is infinitely divisible.

### 1.1 Surplus Process

Assuming that the insurer operates two different lines of business and that there is a sparse dependency between these two lines of business, the surplus process is as follows:

$$\mathrm{R(t)} = x_0 + ct - \left(\sum_{i=1}^{N(t)} X_i + \sum_{i=1}^{N^p(t)} Y_i\right).\#(1)$$

where $\{X_i, i \geq 1\}$ is independently and identically distributed in $F_X(\cdot)$,$E(X) = \mu_X > 0$, $E(X^2) = \sigma_X^2$, as the claim amount of the first class of business, $\{Y_i, i \geq 1\}$ is independently and identically distributed in $F_Y(\cdot)$,$E(Y) = \mu_Y > 0$, $E(Y^2) = \sigma_Y^2$, as the claim amount of the second class of business. the claim amount of the second type of business. N(t)denotes the conforming Poisson process with parameter c denotes the premium of the insurance company by the expected value premium there are $c = (1+\theta_1)\lambda\mu_X + (1+\theta_2)\lambda p\mu_Y$.$\theta_1 > 0$,$\theta_2 > 0$.

### 1.2 Proportional Reinsurance

Assuming that the insurer diversifies the claim risk by purchasing proportional reinsurance, and let $q_1(t)$,$q_2(t)$ be the insurer's retention ratio, the claim after the insurer purchases reinsurance is:
$q_1(t)X_i$ ,$q_2(t)Y_i$ then the reinsurance fee is $\delta(q_1(t), q_2(t)) = (1+\eta_1)(1-q_1(t))\lambda\mu_X + (1+\eta_2)(1-q_2(t))\lambda p\mu_Y$.According to Grandell(1991)[8],the claims process can be diffusely approximated as:

$$d\sum_{i=1}^{N(t)} X_i = \lambda E(X_i)dt - \gamma_1 dW_X(t), \gamma_1 = \sqrt{\lambda E(X_I^2)}$$

$$d\sum_{i=1}^{N^p(t)} Y_i = \lambda E(Y_i)dt - \gamma_2 dW_Y(t), \gamma_2 = \sqrt{\lambda E(Y_I^2)}$$

The correlation coefficient of $W_X(t)W_Y(t)$is $\hat{\rho} = \frac{\lambda p}{\gamma_1\gamma_2}E(X_i)E(Y_i)$.Then the wealth process of the insurer after joining the reinsurance is:

$$dX^{q_1,q_2} = [\lambda\mu_X(\theta_1 - \eta_1 + \eta_1 q_1(t)) + \lambda p\mu_Y(\theta_2 - \eta_2$$
$$+\eta_2 q_2(t))dt + \sqrt{(q_1\gamma_1)^2 + (q_2\gamma_2)^2 + 2\hat{\rho}q_1q_2\gamma_1\gamma_2}dW_0$$

### 1.3 Financial Market

Suppose the financial market consists of three assets: risk-free assets, stocks, and corporate bonds, and the price processes of the three assets are as follows: The price process of risk-free bonds is given by：$dR(t) = rR(t)dt$.The stock price process S(t) obeys the Heston stochastic volatility model:

$$\begin{cases} dS(t) = S(t)[r + \alpha L(t)dt + \sqrt{L(t)}dW_1(t)], S(0) = s_0 \\ dL(t) = k(\omega - L(t))dt + \sigma\sqrt{L(t)}dW_2(t), \quad L(0) = l_0 \end{cases}$$

R is the risk-free rate,$\alpha$, $k$, $\sigma$, are positive constant.$E[W_1W_2] = \rho t$, $2k\omega \geq \sigma^2$
Following Bielecki (2007)[18], the credit bond price process $p(t, T_1)$, under a realistic measure P, using an approximate model to portray default risk is as follows:

$$dp(t, T_1) = p(t-, T_1)[(r + (1 - H(t))\delta(1-\Delta)dt$$
$$-(1 - H(t-))\zeta dM^p(t)]$$

Where $M^p(t) = H(t) - h^Q\int_0^t \Delta(1 - H(u))du$ is a$g$ − harness， $\delta = h^Q\zeta$ is the credit spread.

### 1.4 Robust optimization problem

Assuming the insurer adopts a reinsurance investment strategy$\varepsilon(t) = (q_1(t), q_2(t), \pi(t), \beta(t))$， $q_1(t), q_2(t)$are the reinsurance strategies adopted by the insurer at moment t in the first and second asset classes, respectively.$\pi(t)$ for the insurance company's investment in equities at time t.，$\beta(t)$for the insurance company's investment in credit bonds at time t. Let $\mathbb{A}$ denotes the set of all feasible strategies, then the dynamic process of wealth of the insurance company at this moment $X^\varepsilon(t)$ is:

$$dX^\varepsilon(t) = \pi(t)\frac{dS(t)}{S(t)} + \beta(t)\frac{dp(t)}{p(t)} + (X^\varepsilon(t) - \pi(t) - \beta(t))\frac{dB(t)}{B(t)}$$

$$+dX^m(t), X^{\varepsilon}(0) = x_0$$

Assume that the insurer maximizes the terminal T moment expected utility in the financial market, the portfolio index utility, which takes the form of:

$$V(X(T)) = -\frac{1}{\gamma}e^{-\gamma X(T)}$$

Where $\gamma > 0$ is the ambiguity aversion coefficient of the insurer. The insurer's goal is to find the optimal reinsurance-investment strategy $\varepsilon^*(t) = (q_1^*(t), q_2^*(t), \pi^*(t), \beta^*(t))$ to maximizing the expectation of end-use wealth utility for insurers. The objective function of the insurer is:

$$V^{\varepsilon}(t,x,l,h) = E(u(X^{\varepsilon}(T))|X^{\varepsilon}(t) = x, L(t) = l)$$

The value function of the optimization problem is:

$$V(t,x,l,h) = \sup_{\varepsilon\in\Pi} V^{\varepsilon}(t,x,l,h),\ V(T,x,l,h)=v(x),$$

Assume that the ambiguity information is described by the probability P and the reference model is measured by the probability $\mathcal{P}^{\Phi}$ which is equivalent to P：$\mathcal{P} := \{\mathcal{P}^{\Phi}|\mathcal{P}^{\Phi} \sim P\}$.Next, the optional measure set is constructed, defining the procedure :$\Phi(t) = (\Phi_0(t), \Phi_1(t), \Phi_2(t), \Phi_3(t))$s.t.:

1. $\Phi(t)$ is a $\mathcal{g}(t)$- measurable for any t[0,T]
2. $\Phi_i(t) = \Phi_i(t,\omega), i = 0,1,2,3$ and $\Phi_i(t) \geq 0$ for all $(t,\omega) \in [0,T] \times \Omega$.
3. $\int_0^T ||\Phi(t)||^2 dt < \infty$;

Let $\Sigma$ be all processes shaped as $\Phi$，for all $\Phi \in \Sigma$ define a G-adaptation process under a real measure $\{\Lambda^{\Phi}(t)|t \in [0,T]\}$.From Ito differentiation we have:

$$d\Lambda^{\Phi}(t) = \Lambda^{\Phi}(t-)(-\Phi_0(t)dW_0 - \Phi_1(t)dW_1 - \Phi_2(t)dW_2 - (1-\Phi_3(t))dM^p)$$

Where $\Lambda^{\Phi}(0) = 1$，P-a.s. $\Lambda^{\Phi}(t)$ is a (P,G)- martingale，$E[\Lambda^{\Phi}(T)]$ =1, for each $\Phi \in \Sigma$,A new optional measure is absolutely continuous to P, defined as

$$\frac{dP^{\Phi}}{dP}|_{\mathcal{G}_T} = \Lambda^{\Phi}(T).$$

So far, we have constructed a class of real-world probability measures $P^{\Phi}$，where $\Phi \in \Sigma$，From Gisanova's theorem[21],it follows that:

$$dW_i^{P^{\Phi}}(t) = dW_i(t) + \Phi_i(t)dt, i = 0,1,2$$

Therefore the wealth process in the $P^{\Phi}$ is：

$$\begin{aligned}
dX^{\varepsilon}(t) = [&xr + \pi(\alpha l - \Phi_1\sqrt{l}) + \beta(1-H(t))\delta(1-\Delta) \\
&+\lambda\mu_X(\theta_1 - \eta_1) + \lambda p\mu_Y((\theta_2 - \eta_2) + \eta_1 q_1(t)) + \eta_2 q_2(t)) \\
&+\sqrt{(q_1\gamma_1)^2 + (q_2\gamma_2)^2 + 2\hat{\rho} q_1 q_2\gamma_1\gamma_2}\Phi_0]dt + \pi\sqrt{l}dW_1^{P^{\Phi}} \\
&+\beta(1-H(t-))\zeta dM^p + \sqrt{(q_1\gamma_1)^2 + (q_2\gamma_2)^2 + 2\hat{\rho} q_1 q_2\gamma_1\gamma_2}dW_0^{P^{\Phi}} \quad (2)
\end{aligned}$$

We assume that the insurer determines a robust portfolio strategy such that the worst-case scenario is the best option. The insurer penalizes any deviation from the sub-reference model with a penalty that increases with this deviation, using relative entropy to measure the deviation between the reference measure and the optional measure. Inspired by Maenhout[19] Branger[20] the problem is modified to define the value function as：

$$V(t,x,l.h) = \sup_{\varepsilon\in\Pi}\inf_{\Phi\in\Sigma} E_{t,x,l,h}^{P^{\Phi}}\left[-\frac{1}{\gamma}e^{-\gamma X(T)} + \int_t^T G(u, X^{\varepsilon}(u), \phi(u))du\right]$$

$E_{t,x,l,h}^{P^{\Phi}}$Calculated under the optional measure , the initial value of the stochastic process is $X^{\varepsilon}(t)$=x, S(t)=s, H(t)=h,

$$\begin{aligned}
G(u, X^{\varepsilon}(u), \phi(u)) = &\frac{{\Phi_0}^2}{2\Psi_0(t, X^{\varepsilon}(t), \phi(t))} + \frac{{\Phi_1}^2}{2\Psi_1(t, X^{\varepsilon}(t), \phi(t))} \\
&+\frac{{\Phi_2}^2}{2\Psi_2(t, X^{\varepsilon}(t), \phi(t))} + \frac{(\Phi_3 \ln\Phi_3 - \Phi_3 + 1)h^p(1-h)}{2\Psi_3(t, X^{\varepsilon}(t), \phi(t))}
\end{aligned}$$

Where $\Psi_0 \geq 0$，$\Psi_1 \geq 0$，$\Psi_2 \geq 0$，$\Psi_3 \geq 0$ is state-dependent，let $\Psi_0 = -\frac{v_0}{\gamma V(t,x,l,h)}$, $\Psi_1 = -\frac{v_1}{\gamma V(t,x,l,h)}$, $\Psi_2 = -\frac{v_2}{\gamma V(t,x,l,h)}$, $\Psi_3 = -\frac{v_3}{\gamma V(t,x,l,h)}$

$v_i$,i=0,1,2,3 is the risk aversion factor, the lager $\Psi_i$. According to the dynamic planning principle, the HJB equation is as follows:

$$\sup_{\varepsilon\epsilon\prod}\inf_{\Phi\epsilon\Sigma}\mathcal{A}^{\varepsilon,\phi}V + G(u, X^{\varepsilon}(u), \phi(u)) = 0\#(3)$$

$\mathcal{A}^{\varepsilon,\phi}$is the infinitesimal operator under the measure $P^{\Phi}$.

## III. Robust optimal reinsurance-investment strategy solving

This section will solve the robust optimal problem constructed in the previous section. This paper divides the value function into pre-default and post-default components according to the time of default of the credit bond:

$$V(T,x,l,h)=\begin{cases}V(T,x,l,0),\ h=0(\text{before default})\\ V(T,x,l,1),\ h=1(\text{after default})\end{cases}$$

By decomposing the value function into two sub-functions, denoted as the value function before the zero-coupon bond default and the value function after the zero-coupon bond default, the two sub-HJB equations are obtained and solved successively to obtain the reinsurance and risky asset investment strategies and value function expressions after default, and the reinsurance, risky asset and credit bond investment strategies and value function expressions before default.

### 1.5 Optimal reinsurance and investment decisions after default

When H (t)=1, $\tau\wedge T\le t\le T$,the insurer has constituted a default at or before time t, the HJB equation degenerates to:

$$V_t+\left[rx+\pi\left(\alpha l-\Phi_1\sqrt{l}\right)+\lambda\mu_X(\theta_1-\eta_1)+\lambda p\mu_Y(\theta_2-\eta_2)+\lambda\mu_X\eta_1q_1(t)+\lambda p\mu_Y\eta_2q_2(t)+\sqrt{(q_1\gamma_1)^2+(q_2\gamma_2)^2+2\hat{\rho}q_1q_2\gamma_1\gamma_2}\Phi_0\right]V_x+\frac{1}{2}(\pi^2l+\lambda(q_1\sigma_X)^2+\lambda p(q_2\sigma_Y)^2+2\lambda p\mu_X\mu_Y)V_{xx}+\pi l\sigma\rho V_{xl}+$$

$$(k(\omega-l)-\Phi_1\sigma\rho\sqrt{l}-\Phi_2\sigma\sqrt{1-\rho^2}\sqrt{l})V_l+\frac{1}{2}\sigma^2lV_{ll}-\frac{{\Phi_0}^2}{2v_0}\gamma-\frac{{\Phi_1}^2}{2v_1}\gamma-\frac{{\Phi_2}^2}{2v_2}\gamma=0 \qquad (4)$$

Satisfied: $V(T,x,l,1)=-\frac{1}{\gamma}e^{-\gamma X}$

The solution can be assumed to be of the form:

$$V(t,x,y,l,1)=-\frac{1}{\gamma}\exp\{-\gamma e^{r(T-t)}x+G(t,l)\},G(T,l)=0\#(5)$$

Taking each partial derivative of V:

$$\begin{cases}V_t=\left[\gamma rxe^{r(T-t)}+G_t\right]V,V_x=-\gamma e^{r(T-t)}V\\ V_{xx}=\gamma^2e^{2r(T-t)}V,\ V_{xl}=-\gamma G_lV\\ V_l=G_lV,V_{ll}=G_{ll}V+{G_l}^2V\end{cases}$$

The minimum value point of $\Phi^*$is obtained from the first order condition as:

$$\begin{cases}{\Phi_0}^*=v_0e^{r(T-t)}\sqrt{(q_1\gamma_1)^2+(q_2\gamma_2)^2+2\hat{\rho}q_1q_2\gamma_1\gamma_2}\\ {\Phi_1}^*=v_1e^{r(T-t)}\pi\sqrt{l},{\Phi_2}^*=-\sigma\rho\sqrt{l}G_lv_2\frac{1}{\gamma}\end{cases}$$

Bringing in the HJB equation yields:

$$\gamma rxe^{r(T-t)}+G_t-[\lambda\mu_X(\theta_1-\eta_1)+\lambda p\mu_Y(\theta_2-\eta_2)]$$
$$\gamma e^{r(T-t)}+k(\omega-l)G_l+\frac{\left(\sigma\rho\sqrt{l}G_l\right)^2v_2}{2}+\frac{1}{2}\sigma^2l\left(G_{ll}+{G_l}^2\right)$$
$$+\inf_{\pi}\{f_2(\pi,t)\}+\inf_{q_1,q_2}\{\lambda\gamma e^{r(T-t)}f_1(q_1,q_2,t)\}=0\#(6)$$

Where

$$f_1(q_1,q_2,t)=-[\mu_X\eta_1q_1(t)+p\mu_Y\eta_2q_2(t)]+\frac{(\gamma+v_0)e^{r(T-t)}}{2}[(q_1\sigma_X)^2+p(q_2\sigma_Y)^2+2p\sigma_X\sigma_Yq_1q_2]$$

$$f_2(\pi,t)=\pi le^{r(T-t)}(\sigma\rho G_l(\gamma+v_1)-\alpha)+\frac{1}{2}(\gamma+v_1)l\pi^2e^{2r(T-t)}$$

**Theorem III.1** Let m=$\frac{\mu_X(\eta_1\sigma_Y{}^2-p\eta_2\mu_Y{}^2)}{\sigma_X^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2}$, n=$\frac{\mu_Y(\eta_2\sigma_X{}^2-\eta_1\mu_X{}^2)}{\sigma_X^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2}$,then there exist $t_1,t_2\hat{t_1},\hat{t_2}$ and the following values can be obtained:

$$t_1 = T - \frac{1}{r}\ln\frac{m}{\gamma + v_0}, t_2 = T - \frac{1}{r}\ln\frac{n}{\gamma + v_0}$$
$$\widehat{t_1} = T - \frac{1}{r}\ln\frac{\mu_X\eta_1}{(\gamma + v_0)p(\sigma_X{}^2 + \mu_X\mu_Y)}\ \widehat{t_2} = T - \frac{1}{r}\ln\frac{\mu_y\eta_2}{(\gamma + v_0)(\sigma_Y{}^2 + \mu_X\mu_Y)}$$

When $m \le \gamma(n \le \gamma)$,let $t_2 = T(t_1 = T)$.When $m > \gamma(n > \gamma)$,let $t_2 = 0(t_1 = 0)$(1)If $m \le n$,then $q_1^* \le q_2^*$, for all $t\epsilon[0, T]$, The reinsurance strategy corresponding to the problem is$(q_1^*, q_2^*) = \begin{cases} (\widehat{q_1}, \widehat{q_2}),\ 0 \le t \le t_2 \\ (\widetilde{q_1}, \tilde{1}),\ t_2 \le t \le \widehat{t_1} \\ (1,1),\ t_1 \le t \le T \end{cases}$ (2)If n > m,then for all$t\epsilon[0, T]$, The reinsurance strategy corresponding to the problem is$(q_1^*, q_2^*) = \begin{cases} (\widehat{q_1}, \widehat{q_2}), 0 \le t \le t_1 \\ (1, \widetilde{q_2}), t_1 \le t \le \widehat{t_2} \\ (1,1), \widehat{t_2} \le t \le T \end{cases}$. Where

$$\widehat{q_1} = \frac{\mu_X(\eta_1\sigma_Y{}^2 - p\eta_2\mu_Y{}^2)}{(\gamma + v_0)e^{r(T-t)}(\sigma_x^2\sigma_Y{}^2 - p\mu_X{}^2\mu_Y{}^2)}\#(7)$$
$$\widehat{q_2} = \frac{\mu_Y(\eta_2\sigma_X{}^2 - \eta_1\mu_X{}^2)}{(\gamma + v_0)e^{r(T-t)}(\sigma_x^2\sigma_Y{}^2 - p\mu_X{}^2\mu_Y{}^2)}\#(8)$$
$$\widetilde{q_1} = \frac{\mu_X\eta_1 - p\mu_X\mu_Y(\gamma + v_0)e^{r(T-t)}}{\sigma_X{}^2(\gamma + v_0)e^{r(T-t)}p}\#(9)$$
$$\widetilde{q_2} = \frac{\mu_y\eta_2 - \mu_X\mu_Y(\gamma + v_0)e^{r(T-t)}}{\sigma_Y{}^2(\gamma + v_0)e^{r(T-t)}}\#(10)$$

**Proof:** By finding the first-order partial derivatives, second-order partial derivatives and second-order mixed partial derivatives for$f_1$, the following system of equations and the Hessian array are obtained:

$$\begin{cases} \frac{\partial f_1}{\partial q_1} = -\mu_X\eta_1 + (\gamma + v_0)e^{r(T-t)}[\sigma_X{}^2q_1 + p\sigma_X\sigma_Y q_2] = 0 \\ \frac{\partial f_1}{\partial q_2} = -\mu_Y\eta_2 + (\gamma + v_0)e^{r(T-t)}[\sigma_Y{}^2q_2 + \sigma_X\sigma_Y q_1] = 0 \end{cases} \quad (11)$$

$$\begin{vmatrix} \frac{\partial^2 f_1}{\partial q_1\partial q_1} & \frac{\partial^2 f_1}{\partial q_1\partial q_2} \\ \frac{\partial^2 f_1}{\partial q_2\partial q_1} & \frac{\partial^2 f_1}{\partial q_2\partial q_2} \end{vmatrix} = e^{4r(T-t)}(\gamma + v_0)^2 \begin{vmatrix} -\sigma_x^2(\gamma + v_0) & -p\sigma_X\sigma_Y \\ -p\sigma_X\sigma_Y & -p\sigma_y^2(\gamma + v_0) \end{vmatrix}$$

From the Hessian array positive definite it is known that $f_1(q_1, q_2, t)$ is a convex function and there exist extreme value points; solving the system of equations (11) yields (7). (8).

Obviously, $\widehat{q_1}$、$\widehat{q_2}$ are both increasing functions with respect to t, when $m \le n, 0 \le t \le t_1$ or $n > m$, $0 \le t \le t_2$the solution as (7)(8), Also the values of $t_1$and $t_2$ can be found. When $m \le n$, $t_2 \le t \le \widehat{t_1}$, then $(q_1{}^*, q_2{}^*) = (\widetilde{q_1}, 1), q_1 \in [0,1]$.When $n > m$, $t_1 \le t \le \widehat{t_2}$ then $(q_1{}^*, q_2{}^*) = (1, \widetilde{q_1}), q_2 \in [0,1]$,Separate solve $\frac{df_1(t,q_1,1)}{dq_1} = 0$, $\frac{df_1(t,1,q_2)}{dq_2} = 0$ , then can get $\widetilde{q_1}, \widetilde{q_2}$ as (9), (10) End.

**Theorem III.2** The post-default insurer's optimal risky asset investment strategy is to

$$\pi^* = \frac{\alpha - \sigma\rho(\gamma + v_1)G_1}{(\gamma + v_1)e^{r(T-t)}}\#(12)$$

The expression of the optimal value function is:

$$V(t, x, y, l, 1) = -\frac{1}{\gamma}\exp\{-\gamma e^{r(T-t)}(t)x + G_1(t)l + G_2(t)\}\#(13)$$

*where*
$$\rho \ne \pm 1, G_1 = \frac{l_1 l_2 - l_1 l_2 e^{-\frac{1}{2}\sigma^2(l_1-l_2)(\gamma+v_1)(1-\rho^2)(T-t)}}{l_2 - l_1 e^{-\frac{1}{2}\sigma^2(l_1-l_2)(\gamma+v_1)(1-\rho^2)(T-t)}}, \#(14)$$
$$\rho = 1, G_1 = \frac{\alpha^2}{2(\gamma + v_1)(\alpha\sigma + k)}\left(1 - e^{-(\alpha\sigma+k)(T-t)}\right), \#(15)$$

$$\rho=-1, k\neq\alpha\sigma, G_1=\frac{\alpha^2}{2(\gamma+v_1)(k-\alpha\sigma)}\left(1-e^{(\alpha\sigma-k)(T-t)}\right), \#(16)$$

$$\rho=-1, k=\alpha\sigma, G_1=\frac{\alpha^2}{2(\gamma+v_1)}, \#(17)$$

$$l_{1,2}=\frac{-(\alpha\sigma\rho+k)\pm\sqrt{(\alpha\sigma\rho+k)^2+\alpha^2\sigma^2(1-\rho^2)}}{\gamma\sigma^2(1-\rho^2)}$$

$$G_2=\frac{\gamma}{r}[\lambda(\mu_X(\theta_1-\eta_1)+\lambda p(\mu_Y(\theta_2-\eta_2))][1-e^{r(T-t)}]$$

$$+k\omega\int_t^T G_1(s)ds+\int_t^T \gamma e^{r(T-t)}f_1(q_1{}^*,q_2{}^*,s)ds\ \#(18)$$

Proof: Let

$$G(t,l)=G_1(t)l+G_2(t)\#(19)$$

Bringing equation (19) into equation (6) can be obtained as the following two equations:

$$G_1{}'-\frac{\alpha^2}{2(\gamma+v_1)}-G_1(\alpha\sigma\rho+k)+\frac{1}{2}(\gamma+v_1)\sigma^2G_1{}^2(\rho^2-1)=0\#(20)$$

$$G_2{}'-[\lambda(\mu_X(\theta_1-\eta_1)+\lambda p(\mu_Y(\theta_2-\eta_2)]\gamma e^{r(T-t)}$$
$$+k\omega G_1+\lambda\gamma e^{r(T-t)}f_1(q_1^*,q_2^*,t)=0\#(21)$$

Solving equation (20) yields equations (14)-(17). From equation (21) we get (18), and the specific expression of (18) is discussed in the following cases：

1)When $m\leq n, 0\leq t\leq t_2(n>m, 0\leq t\leq t_1)$

$$G_2=\frac{\gamma}{r}[\lambda(\mu_X(\theta_1-\eta_1)+\lambda p(\mu_Y(\theta_2-\eta_2)][e^{r(T-t)}-1]+\widehat{G_1(t)}$$
$$-\lambda(T-t)\{\mu_X\eta_1\frac{\mu_X(\eta_1\sigma_Y{}^2-p\eta_2\mu_Y{}^2)}{(\sigma_x^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2)}+p\mu_Y\eta_2\frac{\mu_Y(\eta_2\sigma_X{}^2-p\eta_1\mu_X{}^2)}{(\sigma_x^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2)}$$
$$-\frac{1}{2}[\left(\frac{\mu_X\sigma_X(\eta_1\sigma_Y{}^2-p\eta_2\mu_Y{}^2)}{(\sigma_x^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2)}\right)^2+p\left(\frac{\mu_Y\sigma_Y(\eta_2\sigma_X{}^2-p\eta_1\mu_X{}^2)}{(\sigma_x^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2)}\right)^2$$
$$+2p\sigma_X\sigma_Y\frac{\mu_X(\eta_1\sigma_Y{}^2-p\eta_2\mu_Y{}^2)}{(\sigma_x^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2)}\frac{\mu_Y(\eta_2\sigma_X{}^2-p\eta_1\mu_X{}^2)}{(\sigma_x^2\sigma_Y{}^2-p\mu_X{}^2\mu_Y{}^2)}]\}$$

2）When $m\leq n, t_2\leq t\leq\widehat{t_1}$,

$$G_2=\frac{\gamma}{r}[\lambda(\mu_X(\theta_1-\eta_1)+\lambda p(\mu_Y(\theta_2-\eta_2)][e^{r(T-t)}-1]$$
$$+\frac{\lambda\gamma}{(\gamma+v_0)}\left(\frac{\mu_X\eta_1}{\sigma_X p}\right)^2(\frac{1}{2}-p)(T-t)+\lambda(-\frac{\mu_X\eta_1 p\mu_X\mu_Y}{\sigma_X{}^2p}$$
$$-\lambda p\mu_Y\eta_2+\frac{\mu_X\eta_1\mu_X\mu_Y}{\sigma_X}+\sigma_Y\frac{\mu_X\eta_1}{\sigma_X})\frac{\gamma}{r}(e^{r(T-t)}-1)$$
$$+[\left(\frac{\mu_X\mu_Y}{\sigma_X}\right)^2+p\sigma_Y{}^2-2p\sigma_Y\frac{\mu_X\mu_Y}{\sigma_X}]\frac{\lambda\gamma(\gamma+v_0)}{2r}(e^{2r(T-t)}-1)+k\omega\widehat{G_1(t)}$$

3）When $m>n, t_1\leq t\leq\widehat{t_2}$,optimal reinsurance strategy $(q_1{}^*,q_2{}^*)$ =(1,$\widetilde{q_2}$),

$$G_2=\frac{\gamma}{r}[\lambda(\mu_X(\theta_1-\eta_1)+\lambda p(\mu_Y(\theta_2-\eta_2)][e^{r(T-t)}-1]$$
$$+\frac{\lambda\gamma}{(\gamma+v_0)}\left(\frac{\mu_y\eta_2}{\sigma_Y}\right)^2\left(\frac{1}{2p}-1\right)(T-t)+k\omega\widehat{G_1(t)}$$
$$+\lambda\mu_y\eta_2\left(\mu_X\eta_1-p\mu_Y\eta_2\frac{\mu_X\mu_Y}{\sigma_Y{}^2}-\sigma_X\frac{\mu_y\eta_2}{\sigma_Y}+\frac{\mu_y\eta_2\mu_X\mu_Y}{2\sigma_Y{}^2}\right)\frac{\gamma\left(e^{r(T-t)}-1\right)}{r}$$
$$+(\sigma_X{}^2-2p\sigma_X\frac{\mu_X\mu_Y}{\sigma_Y}+p(\frac{\mu_X\mu_Y}{\sigma_Y})^2)\frac{\lambda\gamma(\gamma+v_0)}{4r}(e^{2r(T-t)}-1)$$

4) When $m>n, \widehat{t_2}\leq t\leq T(m\leq n, \widehat{t_1}\leq t\leq T)$, optimal reinsurance strategy $(q_1{}^*,q_2{}^*)$ =(1,1),

$$G_2=\frac{\gamma}{r}[\lambda\mu_X\theta_1+\lambda p\mu_Y\theta_2][e^{r(T-t)}-1]$$

$$+k\omega\widehat{G_1(t)}+\frac{\lambda\gamma(\gamma+v_0)}{4}[(\sigma_X)^2+p(\sigma_Y)^2+2p\sigma_X\sigma_Y](e^{2r(T-t)}-1)$$

Where $\widehat{G_1(t)}=k\omega\int_t^T G_1(s)ds=$

$$l_2k\omega(T-t)-2k\omega(\sigma^2(1-\rho^2)^{-1}$$

$$* ln|\frac{l_1-l_2}{l_1-l_2e^{0.5\sigma^2(l_1-l_2)(1-\rho^2)(\gamma+v_1)(T-t)}}|)，\rho\neq\pm1$$

$$\frac{\alpha^2k\omega}{2(k+\alpha\sigma)(\gamma+v_1)}(T-t)+\frac{1}{2}(\frac{\alpha}{(k+\alpha\sigma)(\gamma+v_1)})^2k\omega(1-e^{(k+\alpha\sigma)(T-t)})，\rho=1$$

$$\frac{\alpha^2k\omega}{2(k-\alpha\sigma)(\gamma+v_1)}(T-t)+\frac{1}{2}(\frac{\alpha}{(k-\alpha\sigma)(\gamma+v_1)})^2k\omega(1-e^{(k-\alpha\sigma)(T-t)})，\rho=-1,k\neq\alpha\sigma$$

$$k\omega\frac{[(T-t)\alpha]^2}{4(\gamma+v_1)}，\rho=-1，k=\alpha\sigma.$$ End.

### 1.6 Optimal reinsurance and investment decisions before default

This section considers the optimal pre-default reinsurance-investment strategy and the value function expression based on the previous section, when H(t)=0, $0\leq t\leq\tau\wedge T$.Let the solution of the default prior value function have the following form:

$$V(T,x,l,0)=-\frac{1}{\gamma}exp\{-\gamma e^{r(T-t)}(t)x+K(t,l)\}\#(22)$$

Satisfying the boundary conditions V(T, x, l, 0)=V(x), K(T, l)=0, the HJB equation is transformed into:

$$\begin{gathered}V_t+[\pi(\alpha l-\Phi_1\sqrt{l})+\beta\delta(1-\Delta)+\lambda\mu_X(\theta_1+\eta_1q_1(t)-1))\\+\lambda p\mu_Y\left(\theta_2+\eta_2(q_2(t)-1)\sqrt{(q_1\gamma_1)^2+(q_2\gamma_2)^2+2\hat{\rho}q_1q_2\gamma_1\gamma_2}\Phi_0\right]V_x\\+\frac{1}{2}((\pi)^2l+\lambda(q_1\sigma_X)^2+\lambda p(q_2\sigma_Y)^2+2\lambda p\sigma_X\sigma_Y)V_{xx}\\+\pi l\sigma\rho V_{xl}+(k(\omega-l)-\Phi_1\sigma\rho\sqrt{l}-\Phi_2\sigma\sqrt{1-\rho^2}\sqrt{l}))V_l+\frac{1}{2}\sigma^2lV_{ll}\\+V\left(e^{\gamma\beta(t)\zeta H_1(t)+K(t,l)-G(t,l)}-1\right)h^p\\-\frac{{\Phi_0}^2}{2v_0}\gamma-\frac{{\Phi_1}^2}{2v_1}\gamma-\frac{{\Phi_2}^2}{2v_2}\gamma-\frac{(\Phi_3\ln\Phi_3-\Phi_3+1)h^p(1-h)}{2v_2}\gamma=0\end{gathered}$$

Similarly find the partial derivative of V:

$$\begin{cases}V_t=\left[\gamma e^{r(T-t)}+K_t'\right]V,V_x=-\gamma e^{r(T-t)}V\\V_{xx}=\gamma^2e^{2r(T-t)}V，V_{xl}=-\gamma e^{r(T-t)}K_lV\\V_l=K_lV,V_{ll}=K_{ll}V+{K_l}^2V\end{cases}$$

Bringing the above expression into the HJB equation and fixing the reinsurance-investment strategy, the minimum point of Φ is obtained according to the first-order condition as:

$$\begin{cases}{\Phi_0}^*=v_0e^{r(T-t)}\sqrt{(q_1\gamma_1)^2+(q_2\gamma_2)^2+2\hat{\rho}q_1q_2\gamma_1\gamma_2}\\{\Phi_1}^*=v_1e^{r(T-t)}\pi\sqrt{l},{\Phi_2}^*=-\sigma\rho\sqrt{l}K_lv_2\frac{1}{\gamma}\\{\Phi_3}^*=exp\{\frac{v_3}{\gamma}\left(e^{-\gamma\beta(t)\zeta e^{r(T-t)}+G_1(t,l)-G(t,l)}-1\right)\}\end{cases}\#(23)$$

After bringing (23) into the HJB equation, by inverting the investment strategy we can obtain:

$$\pi^*=\frac{\alpha-\sigma\rho(\gamma+v_1)K_l}{(\gamma+v_1)e^{r(T-t)}},\beta^*=\frac{\ln\frac{1}{\Delta\Phi_3}+K(t,l)-G(t,l)}{\zeta\gamma e^{r(T-t)}}$$

and obviously the expressions for the reinsurance strategy before and after the bond default are the same, so that $g_1(t,q_1,q_2)=f_1(t,q_1,q_2)=-[\mu_X\eta_1q_1+p\mu_Y\eta_2q_2]$

$$+\frac{1}{2}[(q_1\sigma_X)^2+p(q_2\sigma_Y)^2+p\sigma_X\sigma_Y\mu_X\mu_Y\gamma^2](\gamma+v_0)e^{r(T-t)}$$

Bringing $\varepsilon^*=(\pi^*,\beta^*,{q_1}^*,{q_2}^*)$ into the equation, after finishing, we get:

$$\begin{gathered}[-\gamma re^{r(T-t)}+K_t']-[\lambda\mu_X(\theta_1-\eta_1)\\+\lambda p\mu_Y(\theta_2-\eta_2)]\gamma e^{r(T-t)}+k(\omega-l)K_l+\frac{1}{2}\sigma^2l(K_{ll}+{K_l}^2)\\-\lambda\gamma e^{r(T-t)}g_1({q_1}^*,{q_2}^*,t)-g_2(\pi^*,t)-g_3(\beta^*,t)=0\#(24)\end{gathered}$$

Where

$$g_2(\pi^*, t) = \pi^* l e^{r(T-t)}(\sigma\rho K_l(\gamma + v_1) - \alpha) + \frac{1}{2}(\gamma + v_1) l \pi^{*2} e^{2r(T-t)} \quad (25)$$

$$g_3(\beta^*, t) = -\frac{(\Phi_3 - 1)\gamma h^p}{v_3} - \delta\beta\gamma e^{r(T-t)} \#(26)$$

The derivative of equation (26) with respect to β：

$$\beta^* = \frac{\ln\frac{1}{\Delta\Phi_3} + K(t,l) - G(t,l)}{\zeta\gamma e^{r(T-t)}} \#(27)$$

Bringing it into $\Phi_3{}^*$get : $\frac{h^p}{v_3}\Phi_3(t)\ln\Phi_3(t) + h^p\Phi_3(t) - \frac{\delta}{\zeta} = 0$. The equation has dimension one positive roots $\Phi_3$.

Let $K(t,l) = K_1(t)l + K_2(t)$，satisfy the boundary conditions $K_1(T)=0, K_2(T) = 0$，bring it to HJB equation(24)，eliminating the effect of l on the equation yields two equations:

$$K_1' - kK_1 + \frac{1}{2}\sigma^2 K_1{}^2 - \frac{K_1 - G_1}{\zeta}\delta - (\sigma\rho K_1 + \alpha(\delta - r))\alpha(\delta - r)$$
$$+\frac{1}{2}(\sigma\rho K_1 + \alpha(\delta - r))^2 - (\sigma\rho K_1 + \alpha(\delta - r))\sigma\rho K_1 = 0\#(28)$$

$$K_2' - [\lambda\mu_X(\theta_1 - \eta_1) + \lambda p\mu_Y(\theta_2 - \eta_2)]\gamma e^{r(T-t)} + \lambda p\sigma_X\sigma_Y\gamma^2(e^{r(T-t)})^2$$
$$+k\omega K_1 - \frac{\ln\frac{1}{\Delta\Phi_3} + K_2 - G_2}{\zeta}\delta + \left(1 - \frac{1}{\Delta\Phi_3}\right)h^p - f(t, q_1{}^*, q_2{}^*) = 0\#(29)$$

When $\rho \neq \pm 1$,equation (28) is the first-order RICCATI equation that，from the existence uniqueness of the solution $K_1 = G_1$.Then we solve equation(29):let $I(t)=K_2 - G_2$, Satisfying the boundary condition I(T) = 0, then using (29) and (21) we get：$I' = K_2' - G_2{}' = \frac{\delta}{\zeta}I + \frac{\delta}{\zeta}\ln\frac{1}{\Delta\Phi_3} + \frac{\gamma(\Phi_3 - 1)}{v_3}h^p$,

$I = (\ln\frac{1}{\Delta\Phi_3} + \Delta - 1)e^{-\frac{\delta}{\zeta}(T-t)} - \ln\frac{1}{\Delta\Phi_3} - \Delta + 1$.Thus：

$$K_2 = \left(\ln\frac{1}{\Delta\Phi_3} + \frac{\gamma\Delta(\Phi_3 - 1)}{v_3}\right)e^{-\frac{\delta}{\zeta}(T-t)} - \ln\frac{1}{\Delta\Phi_3} + \frac{\gamma\Delta(\Phi_3 - 1)}{v_3} + G_2(30)$$

**Theorem III.3** The optimal pre-default investment and bond investment strategies are:

$$\pi^* = \frac{\alpha - \sigma\rho(\gamma + v_1)K_l}{(\gamma + v_1)e^{r(T-t)}}, \beta^* = \frac{v_3\ln\frac{1}{\Delta\Phi_3} + \Delta\gamma(\Phi_3 - 1)(e^{-\frac{\delta}{\zeta}(T-t)} - 1)}{v_3\zeta\gamma e^{r(T-t)}}$$

The expression of the value function before default is:

$$V(t,x,l,0) = -\frac{1}{\gamma}\exp\{-\gamma e^{r(T-t)}x + K_1(t)l + K_2(t)\}$$

Where $K_1(t) = G_1(t)$，as the formula (14)-(17)，$K_2(t)$ as the formula (30).

## IV. Parameter Sensitivity analysis

In this chapter, we give several numerical examples to test the effect of model parameters on the optimal strategy. First, to make the parameters more realistic, the model parameters for credit bonds are set as follows, based on the estimates from Berndt (2008)[22] and Collin-Dufrensn & Solnik (2001)[23], which are estimated from market data: $1/\Delta = 2.53, h^Q = 0.013, \zeta = 0.52$.The claim amounts for the first and second categories of insurance respectively meet the parameters of $\lambda_X = 1.5, \lambda_Y = 1$exponential distribution . If no special statement is made, other model parameters are assumed as follows:： $t = 3, T = 10, r = 0.06, \gamma = 4$，$\eta_1 = 2, \eta_2 = 3, p = 0.5, v_0 = v_1 = v_3 = 1, \rho = 1, \sigma = 0.16, k = 2, \alpha = 1.5$.

### 1.7 Influence of parameters on optimal reinsurance strategy

Figure.1-2 represents the variation of the optimal reinsurance strategy with the parameters. where Figure.1 shows the variation of reinsurance strategy with probability p. When p increases, the insurer will reduce the amount of reinsurance retention for class I reinsurance and increase the amount of reinsurance retention for class II reinsurance. figure.2 represents the effect of risk aversion coefficient $\gamma$ on reinsurance strategy, when the risk aversion coefficient is larger, the insurer will reduce the reinsurance strategy and purchase more reinsurance to diversify Claims risk.

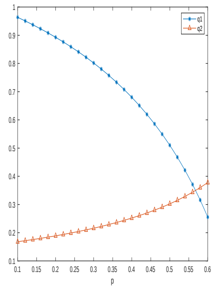

Figure. 1

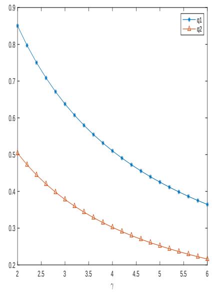

Figure. 2

### 1.8 Influence of parameters on optimal investment strategy

Where Figure.3 represents the effect of the risk-free rate r on the optimal investment strategy$\pi^*$, when the risk-free rate increases, the insurer will invest less in risky assets and more assets in risk-free assets. Figure.4 represents the effect of the volatile reversion rate k on the optimal investment strategy$\pi^*$. When the reversion rate increases, the insurer will increase its investment in risky assets.

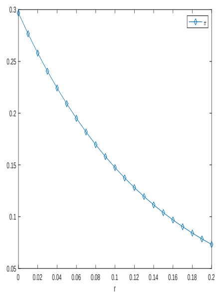

Figure. 3

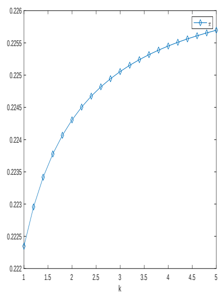

Figure. 4

Figure.5 shows the impact of risk premium $\frac{1}{\Delta}$ on credit bonds, when the risk premium increases, investment in credit bonds is increased. Figure.6 indicates that when the default loss rate $\zeta$ increases, insurers will invest less in credit bond.

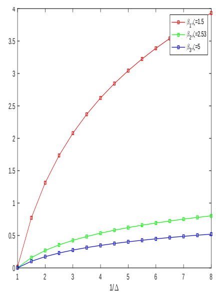

Figure. 5

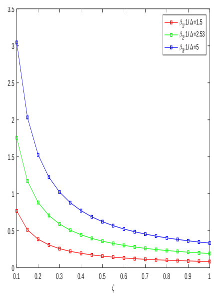

Figure. 6

### 1.9 Influence of ambiguity aversion coefficient on strategy

Figure.7 to Figure.9 represent the effect of optimal reinsurance-investment strategy subject to ambiguity aversion sparsity. It can be seen that as the ambiguity aversion coefficient increases, the insurer's uncertainty about the model increases and therefore reduces its optimal reinsurance-investment strategy.

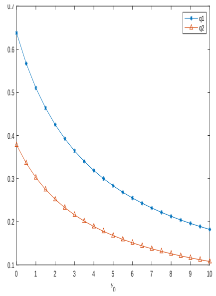

Figure. 7

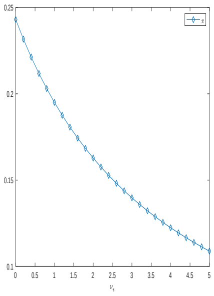

Figure. 8

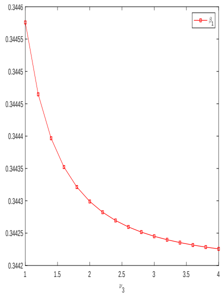

Figure. 9

## V. Conclusion

This paper assumes that the insurer owns two types of insurance business with sparse dependence risk, and the claim process is described by a diffusion approximation model, and secondly, the insurer expands its investment types by investing in the financial market with a stock, a risk-free asset and a credit bond, with the stock price described by a Heston model and the credit bond price described by an approximate model. The credit bond price is described by the approximate model, and considering that the insurer is ambiguous averse, the robust optimal reinsurance-investment problem is established, and the explicit expressions of the robust optimal reinsurance-investment and optimal value function are obtained by using stochastic control theory, dynamic programming principle, and HJB equation, and sensitivity analysis is performed on the model parameters. Based on this paper, further discussions can be made: 1) other situations of the claim process can be considered: such as jump diffusion or common shock. 2) time lag effects can be considered. 3) game problems can be considered.

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