International Journal of Business Marketing and Management (IJBMM)
Volume 4 Issue 10 October 2019, P.P. 26-34
ISSN: 2456-4559
www.ijbmm.com

The Short-term Swap Rate Models in China

Hanhan Guo, Wanxiao Tang, Peibiao Zhao
Department of Applied Mathematics, Nanjing University of Science and Technology, Nanjing 210094, Jiangsu, P. R. China

Abstract: *This paper analyzes the swap rates issued by the China Inter-bank Offered Rate(CHIBOR) and selects the one-year FR007 daily data from January 1st, 2019 to June 30th, 2019 as a sample. To fit the data, we conduct Monte Carlo simulation with several typical continuous short-term swap rate models such as the Merton model, the Vasicek model, the CIR model, etc. These models contain both linear forms and nonlinear forms and each has both drift terms and diffusion terms. After empirical analysis, we obtain the parameter values in Euler-Maruyama scheme and relevant statistical characteristics of each model. The results show that most of the short-term swap rate models can fit the swap rates and reflect the change of trend, while the CKLSO model performs best.*

Keywords: *swap rate models; Euler-Maruyama scheme; Monte Carlo Simulation; swap rates.*

I. Introduction

Short-term interest rate is one of the most important and fundamental economic variables in finance, which determines the structure of interest rates and the price of assets. In Chinese swap market, we have short-term swap rates as a kind of short-term interest rates. Expressed as $dr(t) = f(r(t), t)dt + \sigma(r(t),t)dW_t$, a short-term swap rate model starts from the stochastic differential equation of interest rate and describes the continuous changes during continuous time and continuous state. Classic models like Hull-White model[1], Vasicek model[2], CIR model[3] has been widely used in theoretical modeling and empirical analysis to determine the price and the risk of fixed income assets[4, 5, 6]. Contributing to the flexible form and the advancing statistical methods, short-term interest rates are not limited to the fixed-income market, but widely used in stock prices [7] and stock options[8], insurance[9, 10] and other fields. Can these current models describe the changes in Chinese swap market? Which model performs best in the empirical analysis? They are the problems to be solved in this article.

While in fitting the swap market with the short-term swap rate models, we must make proper parameter estimation. However, the conditional density functions of these stochastic volatility models cannot be explicitly expressed as well as the likelihood functions and unconditional moments. As a result, the parameter estimation is still complicated. Recent studies suggest three methods in parameter estimation. The first method is based on the traditional parameter estimation way by using the approximate methods or the simulated methods to construct the likelihood function and unconditional moment. It mainly include simulating maximum likelihood estimation (like SML), simulating moment estimation (like SMM)[11], etc. The second method indirectly estimates stochastic volatility models by introducing an auxiliary model (such as ARMA, GARCH model)[12, 13] or semi-parametric method[14], which includes indirect inference and effective moment estimation (EMM)[15, 16], etc. The third method is the parameter posterior distribution analysis based on the Bayesian principle[17]. Because of some high dimensional integration, we can hardly calculate the posterior mean and standard deviation. Although scholars overcome the posterior distribution calculation by using the Markov chain

Monte Carlo (MCMC) method, more difficulties appear such as requirements for computing ability and calculation software.

So far, the academic research of short-term interest rate models has achieved fruitful results in both Chinese stock market and bond market interest rate. In this paper, we applies these models to the Chinese swap market. We wonder whether the short-term swap rate models can describe the statistical characteristics in Chinese swap market. As for parameter estimation, since the short-term swap rate models are all stochastic differential equations, we adopt the method issued by Cai Xinrui [18]. Based on the discrete data, this method can derive the distribution of the correlation operation of the numerical solutions in the Euler-Maruyama scheme. Let the data obey this distribution, we can compute the unknown parameters in the drift coefficient and the diffusion coefficient. Based on the algorithm above, we make Monte Carlo simulation with Matlab. As a result, we obtain the parameter values and relevant statistical characteristics along with the most efficient model.

II. Preliminaries

There are many short-term swap rate models with $dr(t) = f(r(t),t)dt + \sigma(r(t),t)dW(t)$ format. The functions $f(\cdot)$ and $\sigma(\cdot)$ are different in specific models. This paper cites nine typical classic short-term swap rate models and improved short-term swap rate models, which is shown in Table 1.

They are widely used in bond pricing, option pricing, and swap pricing. The drift function and the diffusion function are versatile with linear and nonlinear forms.

Table 1: Short-term swap rate models

Number	Model	Expression
1	Merton	$dr = adt + \sigma dW$
2	O-U	$dr = -ardt + \sigma dW$
3	Vasicek	$dr = (ar + b)dt + \sigma dW$
4	CIR	$dr = a(b - r)dt + \sigma\sqrt{r}dW$
5	GBM	$dr = ardt + brdW$
6	CKLSO	$dr = (ar + c)dt + (br + d)dW$
7		$dr = (a - \sigma^2 / 2)dt + \sigma dW$
8		$dr = (-0.5a^2 r)dt + a\sqrt{1 - r^2}dW$
9		$dr = [a^2 r(1 + r^2)]dt + a(1 + r^2)dW$

Consider stochastic differential equation $dr(t) = f(r(\omega,t),\theta)dt + \sigma(r(\omega,t),\gamma)dW(t)$. We denote $r(t_0) = r_0$, $r(t_i) = r_i$, where $t_0 \le t \le T$. ω_i is an element of the underlying probability space Ω. Function $f(\cdot)$ and $\sigma(\cdot)$ are known for given dr(t). W(t) is a standard Brownian motion. In addition, $f(\cdot)$ and $\sigma(\cdot)$ are unknown parameters to be estimated. To simplify the exposition, we assume the parameters α and β are one-dimensional. Moreover, we divide the time interval $[t_0, T]$ into n pieces of equal length h, such that $t_n = T$, $t_n - t_{n-1} = h$, …, $t_2 - t_1 = h$, $t_1 - t_0 = h$.

Discretizing the differential equation above, we obtain the Euler-Maruyama scheme

$$r_{i+1} - r_i = hf(r_i,\alpha) + \sigma(r_i,\beta)[W(t_{i+1}) - W(t_i)] \tag{2.1}$$

Suppose we have m observations $r(\omega_1^{t_0\to t_1}, t_1), r(\omega_2^{t_0\to t_1}, t_1), \ldots, r(\omega_m^{t_0\to t_1}, t_1)$, each appearing after evolution of a time period h from t_0 to t_1 after giving the initial value r_0.

Notice that $r_{i+1} - r_i \sim N(hf(r_i,\alpha), h\sigma^2(r_i,\beta))$ and we have

$$\begin{aligned} &\frac{1}{m}\sum_{j=1}^{m} r(\omega_j^{t_0\to t_1}, t_1) = hf(r_0,\alpha) + r_0 \\ &\frac{1}{m-1}\sum_{j=1}^{m} (r(\omega_j^{t_0\to t_1}, t_1) - hf(r_0))^2 = h\sigma^2(r_0,\beta) \end{aligned} \tag{2.2}$$

According to (2.2), we can estimate $\hat{\alpha}_1$ and $\hat{\beta}_1$.

Similarly, we can define m observations $r(\omega_1^{t_1\to t_2}, t_2), r(\omega_2^{t_1\to t_2}, t_2), \ldots, r(\omega_m^{t_1\to t_2}, t_2)$, each appearing after evolution of a time period h from the given initial value r_1. Then, we have

$$\begin{aligned} &\frac{1}{m}\sum_{j=1}^{m} r(\omega_j^{t_1\to t_2}, t_2) = hf(r_1,\alpha) + r_1 \\ &\frac{1}{m-1}\sum_{j=1}^{m} (r(\omega_j^{t_1\to t_2}, t_2) - hf(r_1))^2 = h\sigma^2(r_1,\beta) \end{aligned} \tag{2.3}$$

According to (2.3), we can estimate $\hat{\alpha}_2$ and $\hat{\beta}_2$.

Repeatedly, we can define m observations $r(\omega_1^{t_i\to t_{i+1}}, t_{i+1}), r(\omega_2^{t_i\to t_{i+1}}, t_{i+1}), \ldots, r(\omega_m^{t_i\to t_{i+1}}, t_{i+1})$, each appearing after evolution of a time period h from the given initial value r_i. Then we can estimate $\hat{\alpha}_3, \hat{\alpha}_4, \ldots, \hat{\alpha}_n$ and $\hat{\beta}_3, \hat{\beta}_4, \ldots, \hat{\beta}_n$.

Finally, we take the average to get the value of the parameters α and β , such that

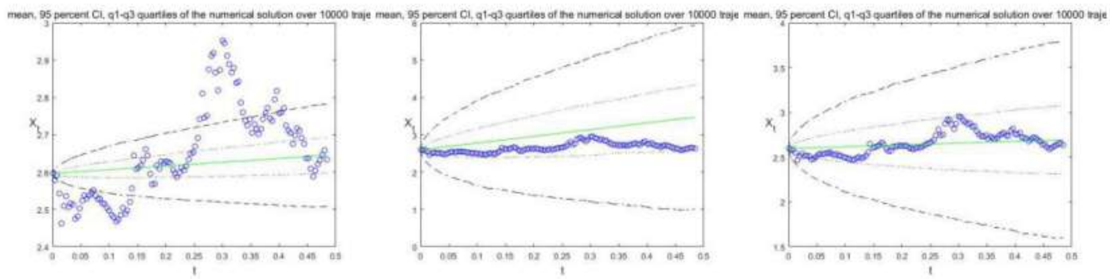
$$\alpha = \frac{1}{n}\sum_{i=1}^{n}\hat{\alpha}_i, \quad \beta = \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}_i \tag{2.4}$$

III. Selection of models and empirical analysis

In this section, we analyze the swap rates compiled and issued by the China Inter-bank Offered Rate (CHIBOR). On the one hand, we fit the swap rates with short-term swap rate models mentioned in section 2. On the other hand, we obtain the parameter values in Euler-Maruyama scheme and relevant statistical characteristics with Monte Carlo Simulation. The sample data comes from the China Money Network. It covers the FR007 fixing curve for 122 working days from January 1st, 2019 to June 30th, 2019. The recording time is 12:00. The price type is average. And the period is one year. First, we divide and serialize the equals time of the data. Generally, the 122 working days in the first half of 2019 will be approximated to 0.5, while the whole year is regarded as unit 1. As a result, in the first working day we have $t_0 = 0$, in the second working day we have $t_1 = 0.004, \cdots$, in the 122nd working day we have $t_{121} = T = 0.484$. Daily swap rate corresponds to its time.

We realize the algorithm with Matlab2016b. During the experiment, we should store the sample data as a '.dat' file, set the trajectories as 10,000 and adjust a smaller step-size of time as 0.001. Before the simulation, we should enter a set of free values for the parameters as 'free parameters' beforehand. Only when the free parameter is reasonable, can the parameter estimation be returned. After running the program, we obtain three figures in total. Figure (a) is a map of 10,000 simulated trajectories. Figure (b) plots original data (∘) vs the empirical mean (green line), the 95% confidence bands (dashed lines) and the first-third quartile (dotted lines) of 10,000 simulated trajectories. Figure (c) is a histogram to show the distribution of 10,000 trajectories.

Note that the parameter estimation will not be returned if the free parameters is unreasonable or the sample data fluctuates greatly. If the free parameters is reasonably, the mean line and the distribution will be similar to the sample data. To illustrate this phenomenon, we exemplify the Merton model (Model 1) and set different free parameters to distinguish 'reasonable free parameters'.



(a) (b) (c)

Figure 1: Contrast between different parameters

In Figure 1(a), we set $a = 0.1, \ \sigma = 0.1$. In Figure 1(b), we set $a = 1.8, \ \sigma = 1.8$. In Figure 1(c), we set $a = 0.2, \ \sigma = 0.8$.

We can observe the original data (∘) vs the empirical mean (green line), the 95% confidence bands (dashed lines) and the first-third quartile (dotted lines) of 10,000 simulated trajectories. Obviously, nearly half of the sample points ran out of the 95% confidence interval in Figure 1(a) while almost all sample points were below the mean value in Figure 1(b). In comparison, the sample points are quite similar to the mean and confidence domains in Figure 1(c). As a result, the free parameters of Figure 1(c) are 'reasonable free parameters'

Table 2: Free parameters of short-term interest rate models

Number	Expression	Free parameters
1	$dr = adt + \sigma dW$	$a = 0.1, \sigma = 1$
2	$dr = -ardt + \sigma dW$	$a = 0.1, \sigma = 1$
3	$dr = (ar + b)dt + \sigma dW$	$a = 0.1, b = 0.1, \sigma = 1$
4	$dr = a(b - r)dt + \sigma\sqrt{r}dW$	$a = -0.1, b = 0.1, \sigma = 0.8$
5	$dr = ardt + brdW$	$a = 0.2, \sigma = 1.2$
6	$dr = (ar + c)dt + (br + d)dW$	$a = 0.15, b = 2, c = 0.5, \sigma = -1$
7	$dr = (a - \sigma^2 / 2)dt + \sigma dW$	$a = 0.25, \sigma = 1.5$
8	$dr = (-0.5a^2 r)dt + a\sqrt{1 - r^2}dW$	$a = 0.4$
9	$dr = [a^2 r(1 + r^2)]dt + a(1 + r^2)dW$	$a = 0.4$

We set the free parameters of short-term swap rate models, which is shown in Table 2. Then, fit the data 10,000 times and we obtain the parameter mean and 95% confidence interval, which is shown in Table 3. The mean of the simulated values of the parameters is equivalent to the estimated value of the parameters. And the last two columns are the 2.5% quantile and the 97.5% quantile, respectively.

Table 3: Parameter Estimation

NO.	Expression	Parameter	Mean	2.50%	97.5%
1	$dr = adt + \sigma dW$	a	0.115441	-1.2248	1.4557
		σ	0.479524	0.4212	0.5378
2	$dr = -ardt + \sigma dW$	a	0.027171	-0.5318	0.4775
		σ	0.479587	0.4213	0.5379
3	$dr = (ar + b)dt + \sigma dW$	a	0.110633	-10.431	10.652
		b	0.115675	-27.891	28.122
		σ	0.479775	0.4211	0.5384
4	$dr = a(b - r)dt + \sigma\sqrt{r}dW$	a	-0.042784	-0.7772	0.6916

		b	-0.010413	-54.832	54.812
		σ	0.291128	0.2556	0.3267
5	$dr = ardt + brdW$	a	0.059523	-0.4358	0.5549
		b	0.176920	0.1552	0.1987
6	$dr = (ar + c)dt + (br + d)dW$	a	-3.000000	-3.0000	-3.0000
		b	0.817881	0.6153	1.0205
		c	8.327462	7.2973	9.3577
		d	-1.709720	-2.2337	-1.1857
7		a	0.230301	-1.1095	1.5701
		σ	0.479527	0.4212	0.5379
8	$dr = (-0.5a^2 r)dt + a\sqrt{1-r^2}dW$	a	0.419982	0.4188	0.4212
9	$dr = [a^2 r(1+r^2)]dt + a(1+r^2)dW$	a	0.057446	0.0504	0.0645

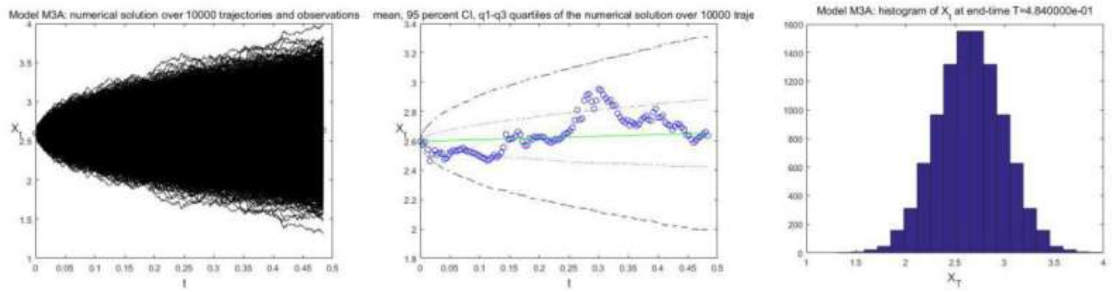
In addition, we also get the statistical characteristics of the swap rate fixing curves with MC simulation shown in Table 4.

Table 4: Statistics characteristics of the swap rate fixing curves

NO.	Expression	Mean	Var.	Std.	Skewness	Kurtosis
1	$dr = adt + \sigma dW$	2.6522	0.1128	0.3359	-1.3522×10^{-13}	2.9553
2	$dr = -ardt + \sigma dW$	2.6307	0.1144	0.3382	-8.5086×10^{-13}	2.9543
3	$dr = (ar + b)dt + \sigma dW$	2.7966	0.1192	0.3453	-2.5583×10^{-14}	2.9513
4	$dr = a(b - r)dt + \sigma\sqrt{r}dW$	2.6510	0.1114	0.3337	0.1842	2.9992
5	$dr = ardt + brdW$	2.6725	0.1105	0.3324	0.3658	3.1924
6	$dr = (ar + c)dt + (br + d)dW$	2.7345	0.0446	0.2112	1.4085	7.0328
7	$dr = (a - \sigma^2/2)dt + \sigma dW$	2.6521	0.1128	0.3359	1.1948×10^{-13}	2.9553
8	$dr = (-0.5a^2 r)dt + a\sqrt{1-r^2}dW$	—	—	—	—	—
9	$dr = [a^2 r(1+r^2)]dt + a(1+r^2)dW$	2.6501	0.1066	0.3265	0.6729	3.9116

As we can see, except for Model 8, the rest models can describe the statistical characteristics of the swap rates. The reason why Model 8 failed is the actual value of swap rate r. In our sample, the swap rate r is always larger than 2, making the fluctuation term an imaginary number which exceeds our calculation range. Hence, its trajectories cannot be plotted on the real number field.

We focus on the parameter estimations among the short-term swap rate models that fit the swap rates successfully. Considering the standard deviation we find Model 6 has the best fitting effect because its standard deviation is the smallest. In Model 6, both the drift function and the diffusion function are linear, which implies the linear form may be good enough to fit the swap rates. In fact, Model 6 also fits better in the bond market than any other models, perhaps more links are worth exploring. As for skewness, Model 1, Model 2, Model 3, and Model 7 exhibit a normal distribution with almost unbiasedness. Model 4, Model 5, Model 9 and Model 6 exhibit a slightly right-biased distribution and the degree became higher. As for kurtosis, model 6 is slightly steep while others are relatively flat.



(a) (b) (c)

Figure 2: Results of Monte Carlo simulation(MC)

We take Model 6 as an example to display more statistical characteristics from Monte Carlo simulation. Figure 2(a) is a map of 10,000 simulated trajectories. Figure 2(b) plots original data (◦) vs the empirical mean (green line), the 95% confidence bands (dashed lines) and the first-third quartile (dotted lines) of 10,000 simulated trajectories. Figure 2(c) is a histogram to show the distribution. We can see the swap rates are growing gradually and fluctuating around 2.6. Besides, shown as Figure 2(c), most trajectories are distributed between 2.6 and 2.7 and the distribution is approximately normal. According to the the empirical mean, we find the overall trend of swap rates is increasing, though the increase is tiny.

In fact, except for Model 8, the rest models can describe the statistical characteristics of the swap rates well. Further more, the fitting results are quite similar. The approximate results may attribute to the stability of Chinese swap rate trend (compared with bond and stock). As we can see, the data has fluctuation but not large, more complicated drift functions and the diffusion functions may not be required. Therefore, in the financial stochastic analysis, especially in swap market, we can select a short-term swap rate model with a simple form. In the way, we are more likely to get a precise empirical result even an analytic solution.

IV. Conclusions

We analyze the swap rates issued by the China Inter-bank Offered Rate(CHIBOR) and select the one-year FR007 daily data from January 1st, 2019 to June 30th, 2019 as a sample. Basing on the data, we conduct Monte Carlo simulation with several typical continuous short-term swap rate models to select the most efficient one. Results are as follows.

(1) Among the tested models, except for model 8, the remaining models can not only fit our sample, but also lead to parameter estimation and statistical characteristics of the swap rates. During the experiment, 'reasonable free parameters' seems the key to parameter estimation.

(2) The empirical mean of Chinese swap rates is relatively horizontal with normally fluctuations.

The overall trend is increasing though the increase is tiny.

(3) Among the tested models that fit the swap rates, Model 6 has the best effect. However, the other models also perform well and the results are quite similar. The approximate results maybe mostly attribute to the stability of Chinese swap rate trend (compared with bond and stock).

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Acknowledgements

This research was supported NNSF of China(No.11371194; No.11501292) and by a Grant-in-Aid for Science Research from Nanjing University of Science and Technology (30920140132035).

References

[1] Hull J, White A. Pricing interest-rate derivative securities. Review of Financial Studies, 1990, 3:573-592 .
[2] Vasicek O. An equilibrium characterization of the term structure. Journal of Financial Eco-nomics, 1977,5:177-188.
[3] Cox J, Ingersoll J, Ross S. A theory of the term structure of interest rates. Econometrica, 1985, 53:385-407.
[4] Xie Chi, Wu Xiong-wei. An empirical analysis of the interest rate behavior in Chinese mon-etary market using the Vasicek and CIR Models. Chinese Journal of Management Science in China, 2002, 10(3):22-25.
[5] Fu Manli, Tu Meizeng, Dong Rongjie. The Vasicek State Space Model and the Empirical Analysis of the Term Structure of the National Bond Interest Rate of the Shanghai Stock Exchange. Systems Engineering-Theory Methodology Application, 2005, 05.
[6] Fan Long-zhen. Empirical analysis of short interest rate models with half-year interest rate in the Shanghai Stock Exchange. Journal of Management Science in China, 2007, 10(2):80-89.
[7] Lin Jianhua, Wang Fuchang, Feng Jinghai. Ornstein-Uhlenbeck process model of stock price. Mathematics In Economic in China, 2000,04:29-32.
[8] Zhou Qing, Li Chao. Pricing Formula of Geometric Mean Asian Options under the Fractional Vasicek Interest Rate Model. Acta Mathematicae Applicatae Sinica, 2014,04:662-675
[9] Wang Bin, Yang Lin, Zhang Shuguang. Optimal Management Under CIR Model. Journal of University of Science and Technology of China, 2003,02, ISSN:0253-2778

[10] Yong-feng Wu, Xue Liang. Vasicek model with mixed-exponential jumps and its applications in finance and insurance. Advances in Difference Equations, Vol.2018(1):1-15
[11] Yuan Jing, Xu dong, Yuwen Jing. An Empirical Analysis of the Sources of Chinese Macroe-conomic Fluctuation. Statistics & Decisions, 2015,02,121-124, ISSN:1002-6487
[12] Fan Zhi, Zhang Shiying. Multi GARCH model and its application in Chinese stock market. Journal of Management Science in China, 2003, 6(2):68-73.
[13] Adn Daz-Hernndez, Nick Constantinou. A multiple regime extension to the Heston Nandi GARCH(1,1) model. Journal of Empirical Finance, 2019,05,004:162-180
[14] Zhang Lin, Yuan Lingxiang. Bayesian Method and Application under Semiparametric Hybrid Model. Statistics & Decisions, 2017,14:84-87, ISSN:1002-6487.
[15] Gallant R, Tauchen G. EMM:A Program for Efficient Method of Moment Estimation. Working Paper, Depart of Economics, University of North Carolina, 2003.
[16] Gao Chi, Guo Yanhong, Yu Yingqi. Dynamic Research on Short-term Interest Rate in China Market-An Empirical Analysis Based on Effective Moment Method. Southern Economy, 2007,04:45-55, ISSN:1000-6249
[17] Zhao Huiqin, Liu Jinshan. Stochastic Volatility Modeling of Shanghai Stock Exchange Us-ing MCMC Method. Journal of Huaqiao University(Natural Science), 2017,02:262-265, ISSN:1000-5013.
[18] Cai Xinrui, Wang Lijin. Some methods of parameter estimation for stochastic differen-tial Equations. Journal of University of Chinese Academy of Sciences, 2017,05:529-537, ISSN:2095-6134